IDENTIFICATION OF PROBABILITY DISTRIBUTIONS FOR FREQUENCY OF FIRE IGNITIONS AND FOR FREQUENCY OF DIFFERENT FIRE SIZES

CLAREMONT GRADUATE SCHOOL

HARVEY MUDD COLLEGE

THE

MATHEMATICS CLINIC

IDENTIFICATION OF PROBABILITY DISTRIBUTIONS FOR FREQUENCY OF FIRE IGNITIONS AND FOR FREQUENCY OF DIFFERENT FIRE SIZES

by

Dennis Duvall
Mehrdad Jalali (Team Leader)
Milton Scritsmier
Dave Smith
Marc Whitney

Prof. Robert Mifflin (Faculty Advisor)
Prof. Robert Borrelli (Consultant)
Prof. Rett Bull (Consultant)

Final Report to

U. S. FOREST SERVICE FIRE LABORATORY

June 1979

ABSTRACT OF CONTENTS

This final report contains the following materials:

*The Management Investment Evaluation Model and our project, which is one component of the overall model, are introduced in Section I.

*Section II discusses problems of analyzing fire data. Related computer programs are listed in Appendix (A).

*Hypothesis testing and different goodness-of-fit tests are discussed in Section III. Related tables are listed in Appendix (E).

*Section IV includes the Poisson process of number of fires, the exponential, Weibull and log-normal distributions for inter-fire times. Also, the results of application of the goodness-of-fit tests are discussed in this section. The computer program and the results of exponential fit can be found in Appendix (B).

*Section V describes the Hazard Function Model for man-caused fires as another approach to the problem.

*The Poisson-Batch Model which has been developed for lightning-caused fires is discussed in Section V. Appendices (C-1), (C-2), (C-3) and (C-4) are related to the problems discussed in the Poisson-Batch Model.

*The breakdown of fires by different characteristics which is a useful output of our study is contained in Appendix (D).

*Finally, Appendix (F) presents the fuel type code list.

NOTE: The Interim Report contains the Individual Fire Report Handbook.

TABLE OF CONTENTS

SECTION I

INTRODUCTION

The Forest Service has set up fire labs in several of the nine regions of the U.S. to conduct research on fire management. These fire labs do some research on their own, and contract out for other projects. Our group works with the Riverside Lab, in region 5. They would like to construct a resource management model, covering all Forest Service lands, in all regions to improve their resource management policy. The research objective of their study is to develope a procedure which incorporates physical fire effects, resource value, fire occurrence and fire management cost and effectiveness within a "cost-benefit" analysis framework. The estimated financial return will be treated as the principle criterion for budget allocation and determination of total justified budget size. Associated information such as the distribution of financial return and the impact on resource output, which is important in fire management decisions will also result from the analytic procedure.

The procedure of the model will be exercised over a broad range of "stylized situations". Each of these situations is defined by given fuel conditions, resource characteristics, fire occurrence levels and weather distributions. Several mixes of fire management activities will be evaluated for each stylized situation.

The output of this model will be an investment guide book which can be used at the regional and national levels to rank alternative fire management activities on the basis of financial returns for various sub-region areas. The analytic results will be separated into two sections in the guide book. The first will be the expected return and the distribution of the financial return about the mean. The second group will be the effect

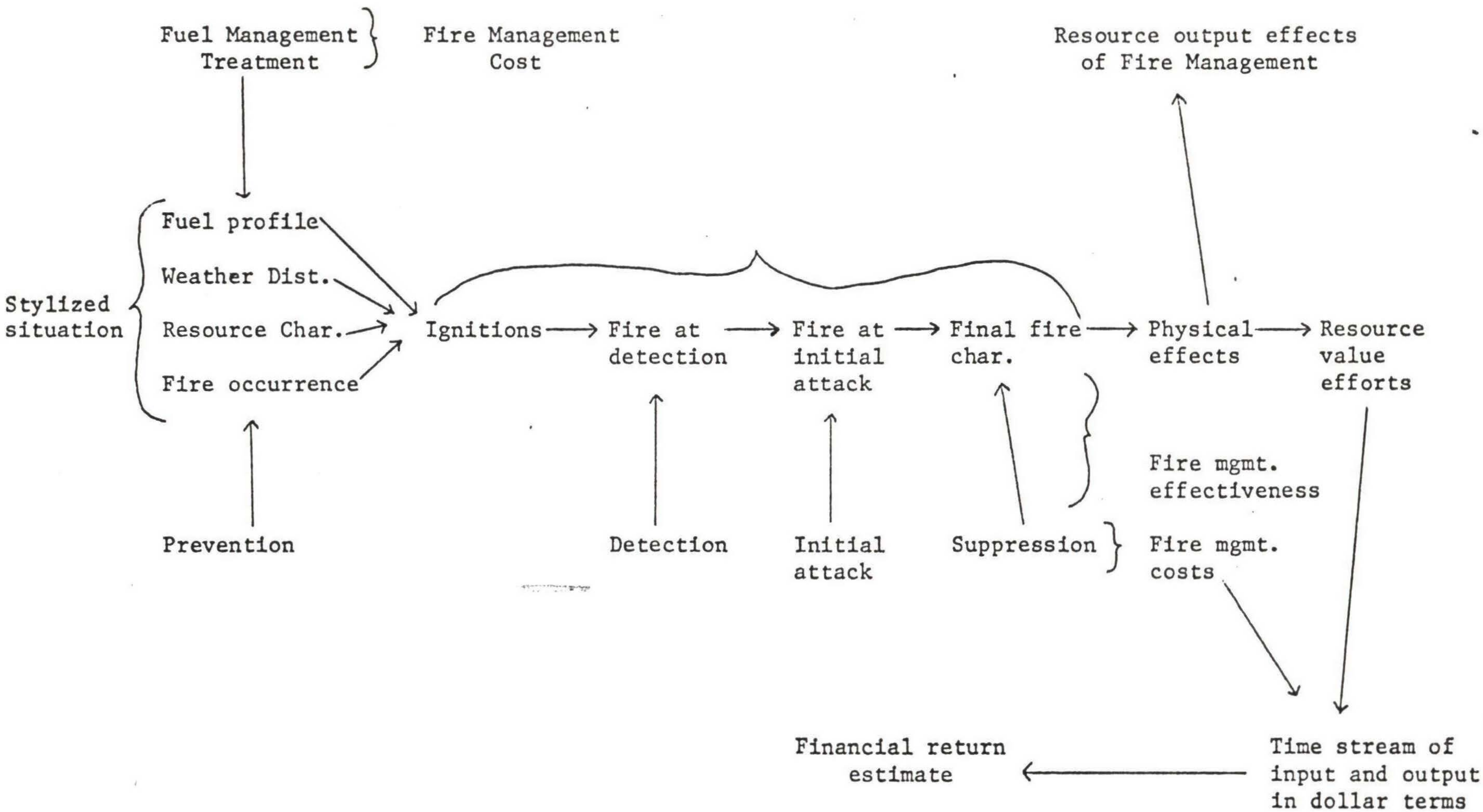
on resource output that will result from applying a particular management activity in a particular stylized situation. The study plan of the Fire Management Investment Evaluation Model can be seen in figure (1).

Our project, determination of probability distributions and goodness-of-fit criteria for frequency of fire ignitions and for frequency of different fire sizes, is one component of the mentioned overall model. One of the usable outputs is the mean and distribution of rate of occurrence of fires by fuel, cause, weather and quarters of the year. Another usable output of the research is the distribution of different fire sizes. These distributions should be stratified by fuel, cause, fire danger and quarter of the year. The frequency of fire occurrence will be per unit forest area and unit time. For reasons of further use of the output in the overall model, the time unit is quarters of a year and the area unit is millions of acres.

In the course of study, first we tried to fit a particular distribution function to a particular empirical distribution (Section III). Since the results were not too satisfactory, we developed two methods which are suggested for further study. These methods are the Hazard Function Model for man-caused fires and the Poisson Batch Model for fires caused by lightning. These methods are discussed in Section IV and V.

FIGUR

SCHEMATIC OF THE FIRE MANAGEMENT INVESTMENT EVALUATION MODEL



SECTION II

DATA ANALYSIS

At the onset of this project the Forest Service provided the first semester clinic team with information about fires in Pacific Coast forest regions over an eight year period from 1970 to 1977. Region number five, consisting of the Angeles, Cleveland, Los Padres, and San Bernardino National Forests, all in Southern California, is of primary interest to the Fire Lab in Riverside, and thus to our analysis. In the eight year period this region accounted for 4,293 reported forest fires.

The fire data was provided in the form of a computer tape, which the first semester team translated from UNIVAC field image to ASCII, a form used in the DEC PDP 1Ø computer at the Claremont Colleges. The translation file, consisting of 4,293 complete records, proved to be too large to store on-line. These records, however, contain many data items which are not of interest to our studies, and we were able to create a smaller data file, one capable of being stored on-line, by extracting out only the useful data items. The original records, for example, contain information about the watershed in the burned area, which is not of direct use to us.

At the same time that the smaller data file was created, the original records were sanitized to identify keypunching errors. Many errors were corrected, but about 50 records from the original file were discarded because critical data was mispunched.

Records in the original file were ordered by a dispatcher code which was assigned when the fire was reported. These codes, unfortunately, were not in chronological sequence, so before we could easily derive the time between two fires for a given stratification it was necessary to order

the new file chronologically. Determining the time between two fires whose occurrence is indicated by a MONTH-DAY-YEAR coding is not a simple problem. To simplify this problem, and to aid in later analysis, the dates within each records were converted to absolute the number of hours since midnight, December 31, 1969. The new data file was then sorted into ascending order keying on this new field, assuring that the correct "time-until-next-fire" could be generated merely by finding the difference between the time sequential records of a given stratification.

With the data in an easily readable form, the first task was to breakdown the records to obtain counts of the number of fires that fall into various stratifications. The results of this breakdown are in Appendix (D).

The fitting of empirical distributions to stratifications of fire characteristics leads to the following problem: the further the data is stratified, the smaller the number of remaining records from which distribution parameters can be estimated. This affects all the confidence bounds on the estimated parameters and the viability of the goodness-of-fit tests which may be applied. Some stratifications, for example the occurrence of lightning fires in the first quarter of the year, have no observations within the data. The number of observations within other stratifications is not large enough to estimate parameters or test the fit of the data to use empirical distribution with any degree of confidence.

One major ramification of this problem is that the estimation of parameters for fires stratified by size becomes difficult once the smaller (1 acre and less) fires are removed. The occurrence of a large fire is a relatively rare event, which is perhaps more a function of the

conditions at the time of ignition than of some distribution describing the time between occurrences of fires. We have been able to side step this problem, however, because the Forest Service is interested in "time until next fire" distributions which are independent of fire size, leaving the distributions for fire sizes, given that a fire has occurred, as a secondary consideration.

Our team chose to approach the fire data in two ways. First, we would consider the problem of finding empirical distributions for the time between fires of a given type. Given the stratification criteria, the inter-fire times could easily be generated directly from the data, which had been sorted into ascending chronological order by the absolute starting hour of the fire. The arrangement of the data also expedited the second approach; modeling the empirical rate at which fires of a given type occur within the data. The first approach concerns itself with estimating the expected time until next fire occurs, while the second approach deals wih estimating the number of fires that can be expected within a given length of time.

To simplify the writing of computer programs to analyze the data, a program called SELECT was written to generate the inter-fire times, given user-specified stratification criteria. SELECT takes the compacted fire-data file as input, and generates fire output files; four quarterly files containing the inter-fire times for fires which started in the quarter, and one file containing only the starting time of the fire (in hours since December 30, 1969) to be used in rate analysis.

SELECT was written in a structured FORTRAN preprocesser named ALTRAN, which is available at Harvey Mudd College. A listing of SELECT appears in Appendix (A).

Before actually starting either of the above approaches, our team tried to get an intuitive feel for the data by producing histograms and graphs of the data, stratified only on cause of fire (lightning or man). Employing several local statistics and graphics programs, we noted several interesting features within the data.

First, inspection of histograms of inter-fire times caused by man showed that the distribution appeared, roughly exponential, with small peaks interrupting the smooth decay expected of an exponential distribution. Further examination showed that these peaks were spaced 24 hours apart. We believe that this is due to the fact that man follows a cylic 24 hour pattern, and as a result starts fires only at certain times of the day, such as at breakfast, lunch, and dinner. Under the assumption of an exponential distribution, it is most likely that the time between fires be less than a day, as for example in the case of one forest fire starting at breakfast time and the next forest fire starting at lunch time of the same day. However, because the times at which man is likely to use fire are concentrated into several fixed periods of the day, it is also likely for example for one forest fire to start at breakfast time of one day and the next forest fire to start at breakfast time several days later, giving an inter-fire time which is a multiple of 24 hours. This latter effect is what we believe causes the 24 hour peaks in the histogram.

SECTION III

STATISTICAL INFERENCE

Parametric Point Estimation

The problem of estimation is defined as follows: Assume that some characteristic of the elements in a population can be represented by a random variable X whose density is $f(.,\theta)$, where the form of the density is assumed to be known except that it contains an unknown parameter θ. Also assume that the values $X_1, X_2, \ldots, X_n$ of a random sample $X_1, X_2, \ldots, X_n$ from $f(.,\theta)$ are observed. On the basis of the observed sample values it is desired to estimate the value of the unknown parameter θ or the value of some function, $\tau(\theta)$, of the unknown parameter. One way of estimation is "point estimation", which is to let the value of some statistic, say $t(X_1, \ldots, X_n)$ represent, or estimate, the unknown $\tau(\theta)$. Such a statistic is called a "point estimation". In our case of study we used the Maximum Likelihood Estimation and the Method of Moments for point estimation of the unknown parameters.

Method of Maximum Likelihood

The likelihood function of random variables $(X_1, X_2, \ldots, X_n)$, $L(\theta_1, \ldots,\theta_k, X_1, \ldots, X_n)$ is the joint density of these variables, $f(X_1, \ldots, X_n, \theta_1, \ldots, \theta_k)$ in which $\theta_1, \ldots, \theta_k$ are the unknown parameters. If $X_1, \ldots, X_n$ is a random sample from the density $f(X; \theta_1, \ldots,\theta_k)$, then:

$$L(\theta_1, \ldots, \theta_k) = f(X_1; \theta_1, \ldots, \theta_k)\cdot f(X_2; \theta_1, \ldots, \theta_k)\cdot f(X_n;\theta_1, \ldots\theta_k) \cdot\cdot$$

Now if $\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k$, the estimators of $\theta_1, \theta_2, \ldots, \theta_k$ which are functions of observations $X_1, \ldots, X_n$, are values that maximize $L(\theta_1, \ldots, \theta_k)$, then $\hat{\theta}_1 = \hat{\theta}_1(X_1, \ldots, X_n), \ldots, \hat{\theta}_k = \hat{\theta}_k(X_1, \ldots, X_n)$ are the "maximum likelihood estimators" of $\theta_1, \ldots, \theta_k$ for the sample $X_1, \ldots, X_n$. If certain regularity conditions hold, the point where the likelihood function is maximized is a solution to the following system of equations:

$$\frac{\partial L(\theta_1, \ldots, \theta_k)}{\partial \theta_1} = 0$$

$$\vdots$$

$$\frac{\partial L(\theta_1, \ldots, \theta_k)}{\partial \theta_k} = 0$$

In our analysis, $X_1, \ldots, X_n$ are random samples from $f(X, \theta_1, \ldots, \theta_k)$, so: $L(\theta_1, \ldots, \theta_k) = f(X_1; \theta_1, \ldots, \theta_k) \cdots\cdot f(X_n; \theta_1, \ldots, \theta_k) = \prod_{i=1}^{n} f(X_i, \theta_1, \ldots, \theta_k)$. Since $L(\theta_1, \ldots, \theta_k)$ is maximized when $\log L(\theta_1, \ldots, \theta_k)$ is maximized, it is easier to solve the following system of equations:

$$\frac{\partial \log L(\theta_1, \ldots, \theta_k)}{\partial \theta_1} = 0$$

$$\frac{\partial \log L(\theta_1, \ldots, \theta_k)}{\partial \theta_k} = 0$$

in which $\log L(\theta_1, \ldots, \theta_k) = \log \prod_{i=1}^{n} f(X_i, \theta_1, \ldots, \theta_k) = \sum_{i=1}^{n} \log f(X_i, \theta_1, \ldots, \theta_k)$

Methods of Moments

Let μ'_r denote the r^{th} moment about 0, i.e. $\mu'_r = E[X^r]$ (expected

value of X^r). μ'_r will be a known function of the k parameters of $f(X, \theta_1, \ldots, \theta_k)$. So, $\mu'_r = \mu'_r(\theta_1, \ldots, \theta_k)$. Let $X_1, X_2, \ldots, X_n$ be random samples from $f(X, \theta_1, \ldots, \theta_k)$ and $M'_j = 1/n \sum_{i=1} X_i^j$ (the j^{th} sample moment).

Now we can form the K equations: $M'_j = \mu'_j(\theta_1, \ldots, \theta_k)(j = 1, \ldots, k)$. Assume $\hat{\theta}_1, \ldots, \hat{\theta}_k$ are the solution to the mentioned system. In this case we say that $(\hat{\theta}_1, \ldots, \hat{\theta}_k)$ is the estimation of $(\theta_1, \ldots, \theta_k)$ obtained by the method of moments.

Goodness of Fit Tests

The use of graphical plotting procedures to test the goodness-of-fit of the data to the proposed distribution works well if the assumed distribution is completely inappropriate or if the data plot nearly perfectly into a straight line. Since subjective judgement must be used, it is often difficult in less clear cut cases to decide whether or not to reject the hypothesized distribution. So we have to use the concept of hypothesis testing.

Hypothesis Testing

A "statistical hypothesis" (H) is an assertion about the distribution of one or more random variables. If the statistical hypothesis completely specifies the distribution, it is called "simple", otherwise, it is called "composite". A "test of statistical hypothesis" (T) is a rule for deciding whether to reject H or not. Now let us define a test (T) of a statistical hypothesis (H) as follows: Reject H if and only if $(X_1, \ldots, X_n) \in C_r$ where $C_r \{(X_1, \ldots, X_n)$: $(X_1, \ldots, X_n)$ is a possible value of $(X_1, \ldots, X_n)\}$. ($X_1, \ldots, X_n$ is a random sample). Then T is

called a "non-randomized" test and C_r is called the "critical region" of the test T. Because of the nature of our study, we are involved with non-randomized tests.

The performance of a non-randomized test is very easy: We should observe a random sample $(X_1, \ldots, X_n)$. If this observation falls into the critical region, we reject H. In many hypothesis testing, like in our case of study, two hypothesis are tested against each other. The first is called the "null hypothesis" (H_o) and the other is called the "alternative hypothesis" (H_1). The relationship between these two is that if H_o is false, the alternative (H_1) is true and vice versa. If the null-hypothesis (H_o) is not rejected, we say H_o is "accepted". In the fire study we want to test if an empirical distribution function, $F_n(X)$ (defined in the next subsection) is the same as, or different from, a proposed distribution $F(X)$. Therefore, we define the null hypothesis and alternative hypothesis in this case as:

$$H_o:\ F_n(X) = F(X)$$

$$H_1:\ F_n(X) \neq F(X)$$

The rejection of H_o when it is true is called a "type I error" and the probability that a type I error is made is called the "size of a type I error", i.e.:

$$\text{Size of a type I error} = \Pr(\text{reject } H_o | H_o \text{ is true}).$$

Acceptance of H_o when it is false is called a "type II error" and its size is the probability that a type II error is made, i.e.:

$$\text{Size of a type II error} = \Pr(\text{accept } H_o | H_o \text{ is false}).$$

The "power function" of a test $(\pi_T(\theta))$ is the probability that H_o is rejected when the distribution from which H_o sample was obtained was parameterized by θ. So: $\pi_T(\theta) = P_\theta$ [reject H_o] $= P_\theta\ (X_1, \ldots, X_n) \in C_r]$. The "size of the test" (T) of H_o is defined to be $\text{Sup}[\pi_T(\theta)]$. By the above definitions it is clear that "size of a type I error" $= \pi_T(\theta)$. In testing we cannot minimize the sizes of type I and type II error simultaneously. So we only minimize the size of type II error which is Pr (accept $H_o | H_o$ is false). By this minimization, we are maximizing the size of type I error which is the power function. One of the test devices that we used in this project is the "Kolmogorov-Smirnov" test.

Kolmogorov-Smirnov Goodness-of-Fit Test

This test can be explained as follows: Given a random sample, $X = (X_1, \ldots, X_n)$ from a continuous distribution function we want to test the "simple" hypothesis that the distribution function is the specified one, i.e. $F(X)$. This is the null hypothesis to indicate that our conclusion would be either "there is enough evidence to reject this hypothesis" or "there is not enough evidence to reject it". The concept of the K-S test is very simple and can be defined as: Let the ordered sample be $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ and let $F_n(X)$ be the sample or empirical distribution function defined by:

$$
\begin{aligned}
F_n(X) &= 0 \quad \text{for } X < X_{(1)} \\
&= i/n \quad \text{for } X_{(i)} \le X < X_{(i+1)} \quad (i = 1, \ldots, n-1) \\
&= 1 \quad \text{for } X_{(n)} \le X
\end{aligned}
$$

If the distribution function of the distribution from which our sample was drawn is really $F(X)$, then this empirical distribution function $F_n(X)$

should be an approximation to $F(X)$. The K-S test uses the following statistic:

$D_n(X) = \underset{\text{all}}{\mathrm{Sup}}|F_n(x) - F(x)|$ and has critical region of the form $\{X: D_n(X) > k\}$, the constant k being chosen to give the desired size of the test (Figure 2). Thus for a size α test, k is chosen so that

$$P_r\{D_n(X) > k|F(x) = \alpha$$

The statistic $D_n(X)$ is said to be "distribution-free". This means that the distribution of $D_n(X)$ and $F(X)$ are independent of each other. We shall not go into the proof of the above statement. The distribution of $D_n(X)$ has been calculated and tabulated for small values of n (sample size). Actually we reject the null hypothesis if $D_n(X)$ is greater than the amount given by the Table for a special size of test. As mentioned before, the K-S test assumes that the null hypothesis is "simple", that is the null hypothesis completely specifies the distribution of the population (no unknown parameters). But in our distribution functions, there are one or more unknown parameters which should be estimated. If there are unknown parameters, $|F_n(X) - F(X,\theta)|$ is no longer a statistic since it depends on θ which is unknown. An obvious way of removing this dependency is to replace θ by an estimator, say $\hat{\theta}$. The test statistic then becomes:

$$\hat{D}_n(X) = \mathrm{Sup}|F_n(x) - F(x,\hat{\theta})| .$$

Application of the K-S Test to the Exponential Distribution Function With Unknown Parameter

The standard tables used for the K-S test are valid when testing whether a set of observations are from a "completely specified" exponential distribution. If λ in $F(x) = \lambda e^{-\lambda x}$ (exponential distribution) is

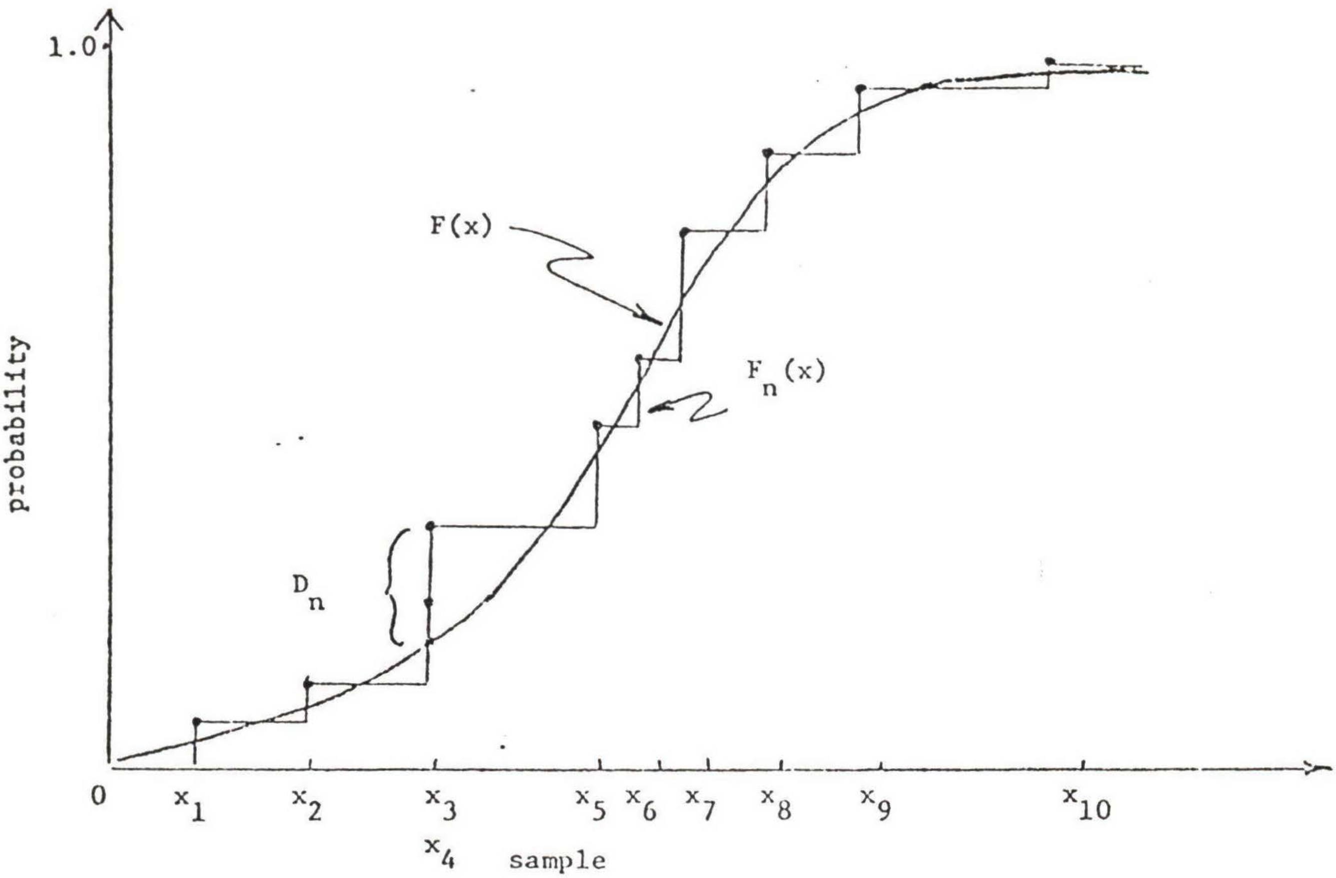


Figure 2: KOLMOGOROV-SMIRNOV TEST

unknown, then we cannot apply the standard table. A new table [6] in this case was obtained by a Monte Carlo calculation. For all odd values of n between 3 and 19 as well as n = 20,25,30 and 35, 5000 samples were drawn and the distribution of D was estimated. Table (2) in Appendix (E) presents the modified critical values of D_n for the exponential distribution. Hence the procedure to apply the test for the exponential distribution is: Given a sample of n observations, determine $D_n = \text{Sup}|F_n(x)-F^*(x)|$, where $F_n(x)$ is the sample distribution function as before and $F^*(x)$ is the cumulative exponential distribution function with $\lambda = \frac{1}{\bar{X}}$ ($\lambda \frac{1}{\bar{X}}$ is the only unknown parameter of the exponential distribution and $\bar{X}$ is the sample mean). If D_n exceeds the critical value in the modified table, reject the hypothesis that the observations are from an exponential distribution with $\lambda = \frac{1}{\bar{X}}$ It is worthwhile to mention that using the standard table would result an extremely conservative test in the sense that the actual significance level, would be much lower than given by the table.

Another test device that is used for exponential distribution in our study, is a test based on the statistic:

$$S^*_n = \sum_{i=1}^{n} |F_n(x_{(i)}) - F^*(x_{(i)})| \text{ , in which } F^*(x) = 1-e^{\frac{-x}{\bar{x}}}$$

This is a test by Finklestein and Schafer which is more powerful than the K-S test in certain cases. Critical values for S^*_n are given by table (3) in Appendix (E). We reject the null hypothesis (the hypothesis that the sample comes from an exponential distribution with $\lambda = \frac{1}{\bar{X}}$), when S^*_n is greater than the value given by the table for a certain significance level.

To test the hypothesis that the proposed distributions are Weibull

($f(x) = \lambda \alpha x^{(\alpha-1)} e^{-\lambda x^{\alpha}}$) with λ and α unknown, we used chi-square test and a test device by Mann-Scheuer and Fertig. The reason for not using the K-S test in the case of Weibull is that there is no special table for the critical values of this test when the proposed distribution is Weibull with unknown parameters. As mentioned before, using the ordinary K-S tables in this case, creates an extremely conservative decision. In the next two sections, the chi-square test and Mann-Scheuer-Fertig test are described.

Chi-Square Goodness-of-Fit Test

Suppose that it is desired to test that a random sample $X_1, \ldots, X_n$ comes from a specific density $f(X, \theta_1, \ldots, \theta_r)$, where $\theta_1, \ldots, \theta_r$ are unknown parameters. The null hypothesis is the "composite" hypothesis, H_o: X_i has density $f(X, \theta_1, \ldots, \theta_r)$ for some value of $\theta_1, \ldots, \theta_r$. If the range of the random variable X_i is decomposed into $k+1$ subsets, say $A_1, \ldots, A_{k+1}$, if $P_i = P[X_i \epsilon A_j]$ and if N_j = number of X_i's falling in A_j, then,

$$\hat{W} = \sum_{1}^{k+1} \frac{(N_j - n\hat{P}_j)^2}{n\hat{P}_j}$$

is approximately distributed as the χ^2 distribution with $(k-r)$ degrees of freedom if n is large and H_o is true, where $\hat{P}_j = P_j(\hat{\theta}_1, \ldots, \hat{\theta}_r)$ and $\hat{\theta}_j$ is a maximum likelihood estimator of θ_j, obtained from the statistics $N_1, \ldots, N_k$. Hence a test of H_o can be obtained by rejecting H_o if and only if the statistic $\hat{W}$ is large, that is, reject H_o "if and only if" the statistic $\hat{W}$ exceeds $\chi^2_{1-\alpha}(k-r)$, where $\chi^2_{1-\alpha}(k-r)$ is the $(1-\alpha)$ th quantile of the χ^2 distribution with $(k-r)$ degrees of freedom. The table of χ^2 distribution is given in Appendix (E).

The K-S test has certain attractive properties, unlike the chi-square

goodness of fit test: (1) the K-S test does not require subjective grouping of data into classes, (2) it is distribution free for all n (sample size), (3) it is consistent in the sense that it has limiting power equal to one.

Mann-Scheuer-Fertig [10] Goodness-of-Fit Test

We used this test in the case of Weibull distribution. This test is based on the properties of adjacent ordered observations.

Consider a random variable X, the natural logarithm of a random variable T. If T is from a two parameter Weibull distribution function, then X has a Type I asymptotic distribution of smallest (extreme) values given by:

$$F(X) = 1 - \exp[-\exp(\frac{x-\eta}{\xi})] , \quad \xi > 0 ,$$

in which η (location parameter) is the mode of the distribution of X and $\frac{\pi \cdot \xi}{\sqrt{6}} = \sigma_X$ (ξ is scale parameter and σ_X is the standard deviation of X).

Now let
$$\ell i = \frac{X_{i+1}-X_i}{E(Z_{i+1})-E(Z_i)} , \quad Z_i = \frac{X_i-\eta}{\xi} .$$

It can be shown that $2\,\ell i$ is asymptotically distributed as chi-square with two degrees of freedom. Then, the statistic

$$W = \frac{\sum_{i=r/2+1}^{r-1} \ell i/[\frac{r-1}{2}]}{\sum_{i=1}^{\frac{r}{2}} \ell i/[\frac{r}{2}]}$$

has approximately an F distribution with $2[\frac{(r-1)}{2}]$ and $2[\frac{r}{2}]$ degrees of freedom (r is the number of observations in which the sample is censored). In order that Monte-Carlo generated critical test values fall in the unit

interval rather than in $(0, \infty)$, the test statistic was transformed to:

$$S = \frac{CW}{1+CW} = \frac{\sum_{i=r/2+1}^{r-1} \ell i}{\sum_{i=1}^{r-1} \ell i} \qquad \text{in which}$$

$$C = \left[\frac{(r-1)}{2} \right] / \left[\frac{r}{2} \right] .$$

S for n sufficiently large or r sufficiently small has approximately a beta distribution with $\frac{r-2}{2}$ and $\frac{r}{2}$ as parameters. Table for sample sizes 3-25 are given by Mann-Scheuer and Fertig (Table (4) in Appendix (E)). We reject the null hypothesis (the hypothesis that the proposed distribution is Weibull) if the statistic S is greater than the value given by the table for a particular level of significance.

SECTION IV

POISSON PROCESS AND INTER-FIRE TIME ANALYSIS

The data was stratified by cause (man-caused and lightning-caused), by fuel type and by quarter of the year. We approached the problem of fitting a distribution to the fires with the assumption that forest fires occur according to a Poisson process $(P(n) = e^{-\lambda} \cdot \frac{\lambda^n}{n!}$ for $n = 0, 1, 2, \ldots)$.

A Poisson process has three major assumptions. Given a sufficiently short time interval of length, h, the probability of one occurrence is approximately λh, where λ is a "constant rate parameter", and the probability of two or more occurrences is approximately zero. Finally, the occurrence of events in non-overlapping time intervals are statistically independent.

The last assumption did seem plausible, since we could assume that the forest was sufficiently large that the occurrence of one fire does not affect the probability of the occurence of another fire. For lightning-caused fires, when a single lightning storm could cause a number of fires, the zero probability of two or more fires at the same time could not be assumed, and another model had to be developed (see Section VI, Poisson-Batch Model).

For the man-caused fires, we continued to apply the Poisson process assumptions. It easily can be shown that if the actual occurrence of events follows a Poisson process, then the time between events has an exponential distribution. That is, if fires occur according to a Poisson process with rate λ fires per unit time then the distribution for "inter-fire" times is exponential distribution with parameter λ, i.e., the probability density is given $f(t) = \lambda \cdot e^{-\lambda t}$ for $t > 0$. For continuous

distributions such as exponential, the K-S type goodness of fit tests are available for testing the hypothesis that the data comes from a proposed distribution (see Section III for details).

The inter-fire times were calculated and plotted on histograms. These histograms showed that a number of the stratifications of fires did not follow an exponential distribution. Therefore, two other distributions, the Weibull $(f(t) = \lambda\alpha \cdot t^{\alpha-1} \cdot e^{-\lambda t})$ and the lognormal $(f(t) = \frac{1}{t\sigma\sqrt{2\pi}} \text{esp} [\frac{-1}{2\sigma^2}(\log t-\mu)^2])$, were selected for study. Both of these distributions have two parameters and therefore allow more freedom to fit the distribution. The lognormal has a "hazard rate" (or fire occurrence rate in our study, which is at first increasing and then decreasing (hazard rate $= \frac{f(t)}{1-F(t)}$. For more detail see Section V).

This would seem to apply during the third quarter where fires occur at a increasing rate early in the quarter, but then taper off and occur at a decreasing rate at the end of the quarter. The Weibull distribution has monotone fire occurrence rate (hazard rate) of $\alpha\lambda \cdot t^{\alpha-1}$ which, it was thought, would apply well during the second and fourth quarters with $\alpha>1$ and $\alpha<1$ respectively.

In the case of the exponential distribution the hazard rate is a constant (λ), i.e. that of the Weibull distribution with $\alpha=1$.

These distributions, the Weibull, the lognormal, and the exponential, were tested with the given data. The parameters were estimated by the maximum likelihood method and validated by the method of moments (for a discfiption of these methods see Section III). After estimating the parameters of the distributions, several goodness-of-fit tests were applied, namely a modified version of the Kolmogorov-Smirov goodness of fit test, The chi-square test and the Mann-Scheuer-Fertig test (for details on these

tests see Section III). With the data on man-caused fires stratified by quarter and by fuel type, these calculations showed that in some cases the exponential fit while in others the Weibull and lognormal fit. (see Table 1)(for the details of the exponential analysis see Appendix B). However, there were a number of stratifications for which none of the tested distributions fit reliably. Some of the problems arose because during the first quarter there were too few fires to apply any statistical methods reliably. In the fuel group consisting of certain types of logging debris, there were no observations in the given data. The failure of the data to fit any of these three distributions in many situations led us to develop other probabilistic models which are presented in the next two sections.

Table 1

MAN-CAUSED FIRES

Fuel Type ↓	Quarter 1	2	3	4
1	Exponential			Log-normal
2				Log-normal
3				Weibull
4				Exponential
5		Weibull	Weibull	Weibull
6	←	No Fire	→	
7	Exponential	Exponential	Exponential	Exponential

The above distributions have been accepted at 0.20 level of significance.

Note: Fuel types are described in Appendix F.

SECTION V

A HAZARD FUNCTION MODEL FOR FIRE IGNITION TIMES OR A NONSTATIONARY POISSON PROCESS FOR NUMBER OF FIRE IGNITIONS

Let the non-negative random variable T be the time until a fire ignition starting from midnight of December 31-January 1 (a time of low fire activity in North American forests). Suppose T has a probability distribution function F with density f and let

$$r(t) = \left(\frac{1}{1-F(t)}\right) \lim_{X \to 0} \frac{F(t+x)-F(t)}{x} = \frac{f(t)}{1-F(t)} .$$

$r(t)$ is called the <u>hazard rate</u> at time t and for Δt small $r(t)\,\Delta t$ is (to within terms of higher order in Δt) the conditional probability of a fire ignition in the time interval given $(t, t+\Delta t)$,

Suppose fire ignitions behave in the following probabilistic manner:

The time until the first fire has distribution $F(\cdot)$. The time between the first and second fires has the distribution $F(\cdot|t_1)$ where t_1 is the time of the first fire and $F(\cdot|\tau)$ is the conditional distribution of $T-\tau$ given $T>\tau$. In general, the time between the $i-1^{st}$ fire and the i^{th} fire has distribution $F(\cdot|t_{i-1})$ where t_{i-1} is the time of the $i-1^{st}$ fire. This assumes that the forest area is so large that the time of the i^{th} fire is not influenced stochasticly by the $i-1$ preceding fires, except for the knowledge that it is greater than t_{i-1}.

It can be shown, using rules of conditional probability, that

$$F(t|t_i) = \frac{F(t+t_i)-F(t_i)}{1-F(t_i)} \qquad \text{for } t \geq 0$$

and, from the definition of the hazard rate, that

$$F(t|t_i) = 1-e^{-\int_{t_i}^{t_i+t} r(x)dx}$$

[Barlow and Proshan; p. 54(1975)].

Let

$$H(t) = \int_0^t r(x)dx$$

so that

$$F(t|t_i) = 1-e^{-(H(t_i+t) - H(t_i))} .$$

$H(\cdot)$ is called the <u>hazard function</u> and, since it is the cummulative ignition rate, $H(t)$ can be interpreted stochasticly as the number of fires in the interval $(0,t)$. In fact, it is shown below that

$$H(t) = E[M(t)]$$

where $M(t)$ is the random number of fires in the interval $(0,t)$ and $E[\cdot]$ denotes expected value. Thus, a "method of moments" estimate of $H(t)$ may be made by estimating $H(\cdot)$ by the empirical counting process defined by

$$N(0) = 0$$

$$N(t_i) = i$$

where t_i is the time of the i^{th} fire in some observed data. $H(\cdot)$ could be estimated by an "S-shaped" approximation to $N(\cdot)$ to reflect the low fire rate in winter and the high fire rate in summer. (Diagram shows the S-shaped cumulative number of fires,) Furthermore; we might assume that $r(\cdot) = \frac{dH}{dt}(\cdot)$ is the same for each year and then estimate H by the average $(N_1+N_2+\ldots+N_k)/k$ where N_j is the empirical counting process

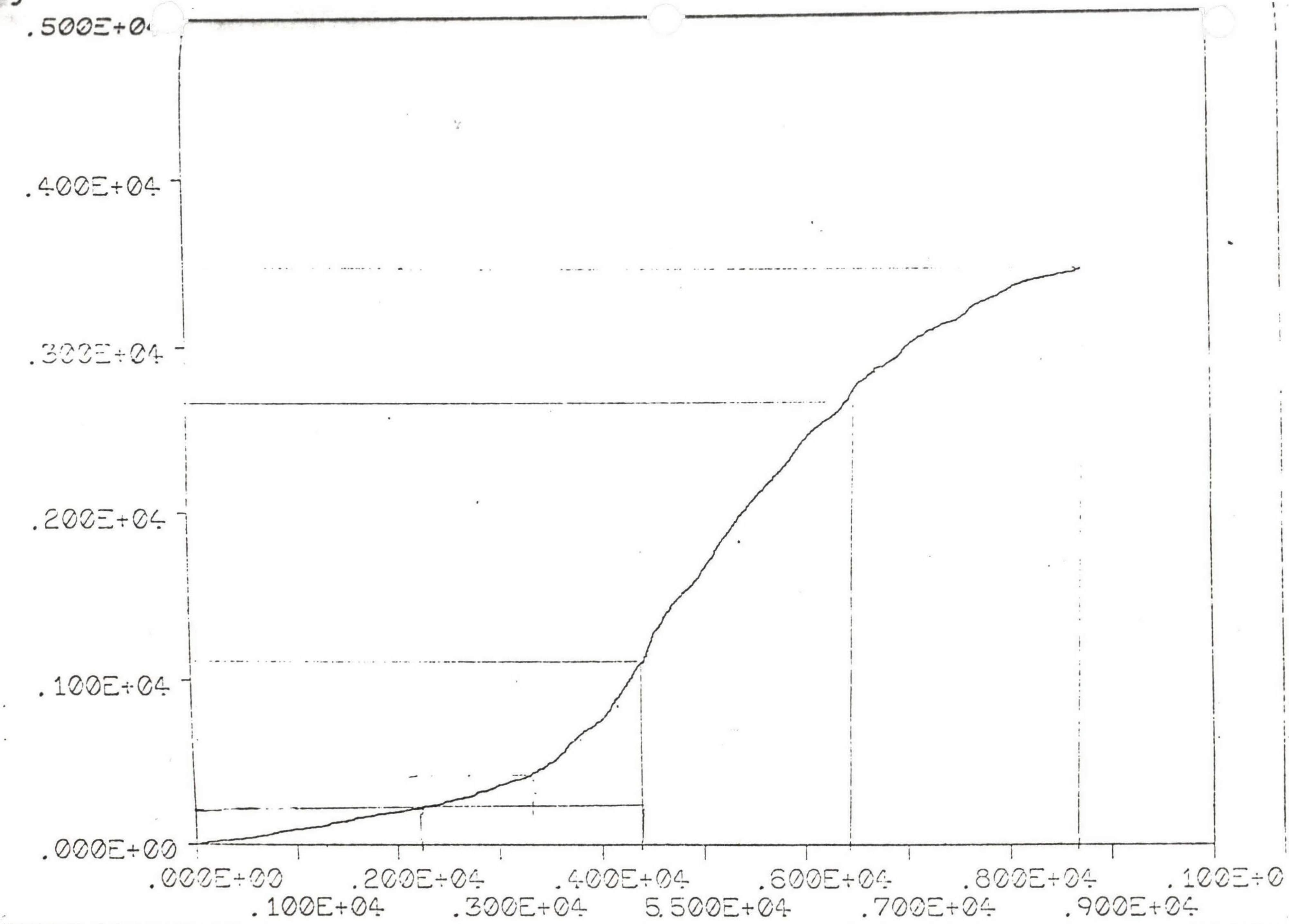

.500E+04
.400E+04
.300E+04
.200E+04
.100E+04
.000E+00
.000E+00
.100E+04
.200E+04
.300E+04
.400E+04
.500E+04
.600E+04
.700E+04
.800E+04
.900E+04

Figure 3

for the j^{th} year out of k years of data. The estimation should be made by a function whose derivative is constrained to have the same value at the beginning of the year as at the end.

Some preliminary Forest Service Region 5 data for man-caused fires shows an average N which is nearly linear in the 1^{st} and 3^{rd} quarters of the year with low and high slopes, respectively, and nonlinear in the 2^{nd} and 4^{th} quarters with positive and negative curvature, respectively. Similar analysis of lightning-caused fires shows that H may not be smooth enough for this model to apply.

In order to calculate the probability mass function for $M(t)$ consider the fire times $T_1, T_2, \ldots$ and the inter-fire times for $i = 0, 1, 2, \ldots$

$$\tau_i = T_i - T_{i-1} \quad \text{where} \quad T_o \equiv 0 \ .$$

Observe that

$$M(t) = 0 \quad \text{if and only if} \quad \tau_1 = T_1 > t$$

and for $m = 1, 2, \ldots$

$$M(t) \geqq m \quad \text{if and only if} \quad T_m = T_{m+1} + \tau_m \leqq t \ .$$

Thus,

$$P[M(t)=0] = P[T_1>t] = P[T>t] = 1-F(t) = e^{-H(t)}$$

and for $m = 1, 2, \ldots$

$$P[M(t)=m] = P[M(t)\geqq m] - P[M(t)\geqq m+1]$$

$$= P[T_m\leqq t] - P[T_{m+1}\leqq t]$$

or letting

$$F_m(t) = P[T_m \leqq t] ,$$

$$P[M(t)=m] = F_m(t) - F_{m+1}(t) \quad .$$

Now, to find a relation between F_{m+1} and F_m , note that

$$F_{m+1}(t) = P[T_{m+1} \leqq t] = P[T_m + \tau_{m+1} \leqq t]$$

or, by conditioning on T_m ,

$$F_{m+1}(t) = \int_o^t P[\tau_{m+1} \leqq t-s | T_m = s] \; dF_m(s)$$

or, by our assumption about the conditional distribution of τ_{m+1} given T_m,

$$F_{m+1}(t) = \int_o^t F(t-s|s) \; dF_m(s)$$

or, in terms of the hazard function,

$$F_{m+1}(t) = \int_o^t [1-e^{-(H(t)-H(s)}] \; dF_m(s)$$

or, by parts integration,

$$F_{m+1}(t) = [[1-e^{-(H(t)-H(s))}] \; F_m(s)]_o^t$$
$$+ \int_o^t F_m(s) e^{-(H(t)-H(s))} dH(s)$$

or, since $F_m(0) = 0$

$$F_{m+1}(t) = e^{-H(t)} \int_o^t F_m(s) e^{H(s)} dH(s) \; .$$

Next we verify that

$$F_m(t) = 1-e^{-H(t)} \sum_{j=o}^{m-1} \frac{(H(t))^j}{j!}$$

satisfies the above equation. With this solution the right hand side is

$$e^{-H(t)} \int_o^t [e^{H(s)} - \sum_{j=o}^{m-1} \frac{(H(s))^j}{j!}] \, dH(s)$$

or, carrying out the integration,

$$e^{-H(t)} [e^{H(s)} - \sum_{j=o}^{m-1} \frac{(H(s))^{j+1}}{(j+1)!}]_o^t$$

or, letting $k = j+1$ and using $H(0) = 0$ in the evaluation,

$$e^{-H(t)} [e^{H(t)} - 1 - \sum_{k=1}^{m} \frac{(H(t))^k}{k!}]$$

which is equivalent to

$$1 - e^{-H(t)} \sum_{k=o}^{m} \frac{(H(t))^k}{k!}$$

and verifies the stated solution for $m+1$.

Thus,

$$P[M(t)=m] = e^{-H(t)} \frac{(H(t))^m}{m!} \qquad \text{for} \quad m = 0, 1, \ldots$$

so, $M(t)$ is a <u>nonstationary Poisson process</u> with parameter equal to the hazard function at t and, hence,

$$E[M(t)] = H(t) \quad .$$

SECTION VI

POISSON BATCH MODEL

The failure of lightning caused fires to fit an exponential distribution was largely due to the fact that groups of the fires were started by the same storm. This dependence of groups of fires upon a single ignition source destroys the assumption necessary for the exponential distribution that each fire be independent of all the other fires. For this reason the Poisson Batch model was devised to explicitly take into account that several fires may depend upon a single source for their ignition. The assumption of a lightning storm as the source of groups of fires is not critical, however, and the model can be extended to any case, such as arson, where several fires depend upon a single source.

The Poisson Batch model assumes that groups (or batches) of potential fires are carried into the forest by a single source (or batch carrier). For lightning-caused fires the batch carrier would be a lightning storm, and the potential fires would be represented by lightning bolts. It is assumed that each batch carrier contains a random number of potential fires which is independent of all batch carriers. One then models the arrival of the batch carriers with a probability distribution which seems consistent with the actual observed arrival times. In the case of lightning caused fires a Poisson distribution with rate parameter λ was used to model the arrival of lightning storms to the forest.

One next models the probability of the potential fires of each batch carrier turning into actual fires by another probability distribution. For the actual number of fires caused by a batch, the Poisson distribution was again used, this time with a (different) parameter μ.

In using the Poisson distribution to model the arrivals of batch carriers, the three assumptions necessary for the Poisson process were made. They are:

(1) There exists a sufficiently small period of time h such that the occurrence of exactly one event is proportional to h, no matter when the time interval h occurs.

(2) Only one event can occur during any time interval h.

(3) The occurrences of events are independent of each other.

It is not clear however, that these assumptions are valid for the arrival of lightning storms over the entire year. In region 5, one expects more lightning storms per time period in late summer than in winter, and this violates assumption (1). For this reason it was decided to fit the Poisson batch model on a quarter by quarter basis, since the occurrence of lightning storms during the time period h is more likely to be constant over a quarter than over a year. Also, groups of lightning storms themselves can be part of even larger weather patterns, and this might destroy the independence assumed in (3). In this case some other discrete probability distribution would have to be assumed for the arrival of the lightning storms, or the model would have to be extended as will be described below.

Once probability distributions have been assumed for the arrival of batch carriers and for the probability of the potential fires of each batch turning into actual fires, the factorial moment generating function $\psi_N(t)$ is derived for N, which is the random variable representing the total number of fires. In the case described above where both distributions are the Poisson distribution, the deriviation in Appendix (C-1) shows that

$$\psi_n(t) = e^{-\lambda+b(t)}$$

where

$$b(t) = \lambda e^{-\mu(1-t)}$$

Derivation of the factorial moment generating function is important, for it allows calculation of the method of moments estimators for λ and μ and, once these have been estimated, the calculation of probabilities. This is also shown in Appendix (C-1).

A chi-square test can then be used to test the Poisson Batch model with estimated parameters λ and μ against the actual observed fires. To do this, a suitable time interval is chosen as a basic unit of time. This time interval should be larger than the time period h mentioned above, but less than a quarter of the year. Since lightning storms seldom last longer than a day, a typical time interval might be a week, a half month or a month. The number of fires that occurred during each time interval entirely within the quarter of the year to be tested is then recorded. This process is repeated for every year for which there is data. Thus if the time interval is a week (with thirteen entire weeks per quarter except for the first quarter) and there are eight years of data, there will be 8 x 13 or 104 time intervals for which the frequency of fire occurrence has been recorded. The number of time intervals which have a given frequency of fires is then counted for each frequency. Typically the number of time intervals which have no lightning caused fires will be relatively high, while the number of time intervals having a greater number of lightning caused fires decreases quite rapidly. Figure 1 shows the distribution of observed fires per week for the third quarter of each of the eight years of available data. Despite the fact that the

distribution is extremely skewed towards 0 fires per week, the third quarter was the most evenly distributed of all four quarters. The first quarter of all eight years, for example, had no lightning fires in them, so that a Poisson Batch model analysis cannot be done for the first quarter. To test the model for the other quarters, a computer program called POSSON was written to sort the fires according to their frequency of occurrence per time interval, to estimate the parameters λ and μ, and to perform the chi-square test. A description of this program, along with a discussion of how the chi-square test is performed, is given in appendix (C-2). The program itself appears in Appendix (C-3).

In Appendix (C-1) it is shown that the probability of n fires occurring per time interval is

$$P[N=0] = e^{-\lambda(1-e^{-\mu})} \qquad \text{for} \quad n = 0$$

$$P[N=n] = \frac{\mu^n}{n!}(P[N=0])\sum_{j=1}^{n} a_{j,n}(\lambda e^{-\mu})^j \qquad \text{for} \quad n > 0$$

By using the values estimated for λ and μ for the third quarter, we can get an idea of what the theoretical probability distribution $P[N=n]$ looks like for the third quarter. It was found that the estimated parameters for the third quarter are

$$\hat{\lambda} = 0.310 \text{ storms/week}$$

$$\hat{\mu} = 19.2 \text{ fires/storm}$$

so inserting these values into the expression for $P[N=n]$ we have

$$P[N=0] = K \qquad \text{for} \qquad n=0$$

$$P[N=n] = K\,\frac{(19.2)^n}{n!} \sum_{j=i}^{n} a_{j,n} b^j \qquad \text{for} \quad n>0$$

where

$$K \equiv \exp\,[-(0.310)\,(1-\exp(-19.2))] \simeq 0.733$$

$$b \equiv (0.310)\,\exp\,(-19.2) \simeq 1.4 \times 10^{-9}$$

are constants. Since $b \simeq 1.4 \times 10^{-9}$, we can to good approximation replace the sum $\sum_{j=1}^{n} a_{j,n} b^j$ by the approximation

$$a_{i,n}\, b = (1)\,(1.4 \times 10^{-9}) = 1.4 \times 10^{-9}$$

since the higher powers of b are very small compared to b and since the coefficients $a_{j,n}$ are not large enough to overcome this effect. Thus we have

$$P[N=0] = K \qquad n=0$$

$$P[N=n] \simeq Kb\,\frac{(19.2)^n}{n!} \qquad n>0$$

Since $b \ll 1$, we see that the Poisson Batch model correctly predicts that $P[N=0]$ will be much greater than $P[N=n]$ for small positive n. Note, however, that until $n \simeq 19$ the expression $\frac{(19.2)^n}{n!}$ will be an increasing function of n since the $(19.2)^n$ term in the numerator will dominate, but that after this point the factorial function in denominator will begin to dominate, causing the expression to steadily decrease. If this expression is evaluated for various values of n, it is found that it does indeed increase steadily until a aximum of about 1.98×10^7 is reached for $n=19$, after which the expression

steadily decreases for $n>19$. Thus a local maximum of $P[N=n]$ occurs at $n=19$, where it is found that $P[N=19] = 0.021$, which is still much less than $P[N=0]$. A plot of $P[N=n]$ versus n is shown in Figure 5. This figure should be compared with Figure 4, since this figure can be interpreted as the empirical relative probability distribution for the third quarter. One finds that while the two histograms are roughly the same, the observed frequency historgram (figure 1) does not decay fast enough immediately after $n=0$ to correspond to figure 2 (for example, from the observed frequencies in figure 1, $P[N=1] \simeq \frac{1}{5} P[N=0]$, while from the calculated probabilities $P[N=1] \simeq (2.7\times10^{-8}) P[N=0]$). Thus it appears that the Poisson Batch model in its present form may not explain the data.

However, our results at this time concerning the Poisson Batch model are inconclusive. As was shown above, the probability derived from the estimated values of μ and λ becomes very small for values of n larger than the estimated value. This causes problems both in calculating the probabilities of of higher frequencies, and in the forming of cells for the chi-square test. These problems are outlined in Appendix (C-2). Greater accuracy in numerical calculation may help, but it might not be possible to obtain a positive or negative result with the present model.

It should be emphasized, however, that the Poisson Batch model can be generalized. Should the present model fail, a new discrete probability distribution can be substituted in place of the Poisson distribution for either the distribution of the arrival of lightning storms or the distribution for the probability of the potential fires turning into actual fires. Also, the problem with the arrival of lightning storms being dependent upon weather patterns might be solved by adding a new level to the Poisson Batch model.

Figure 4: Observed Frequency of Fires per Week for the Third Quarter

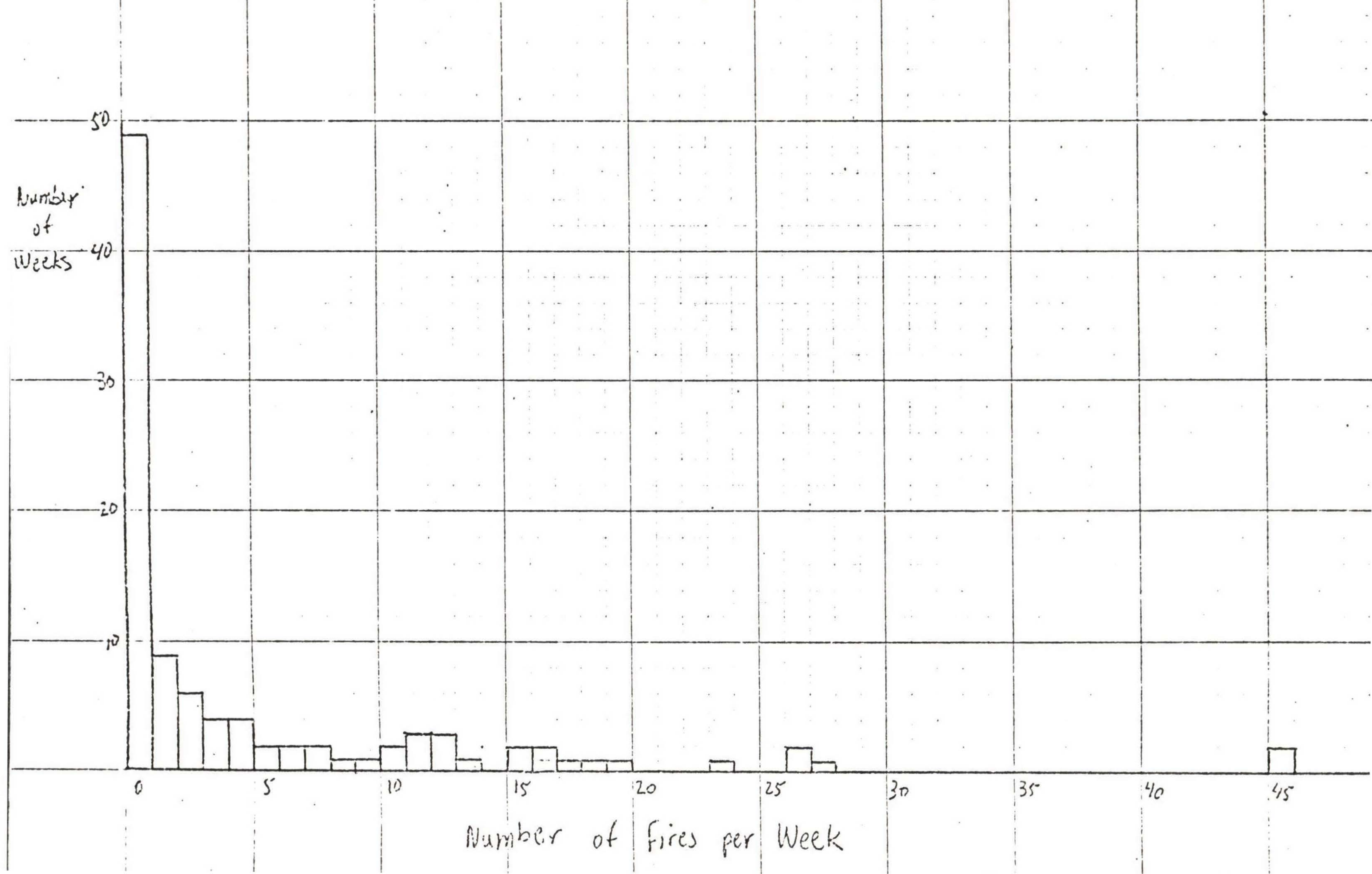
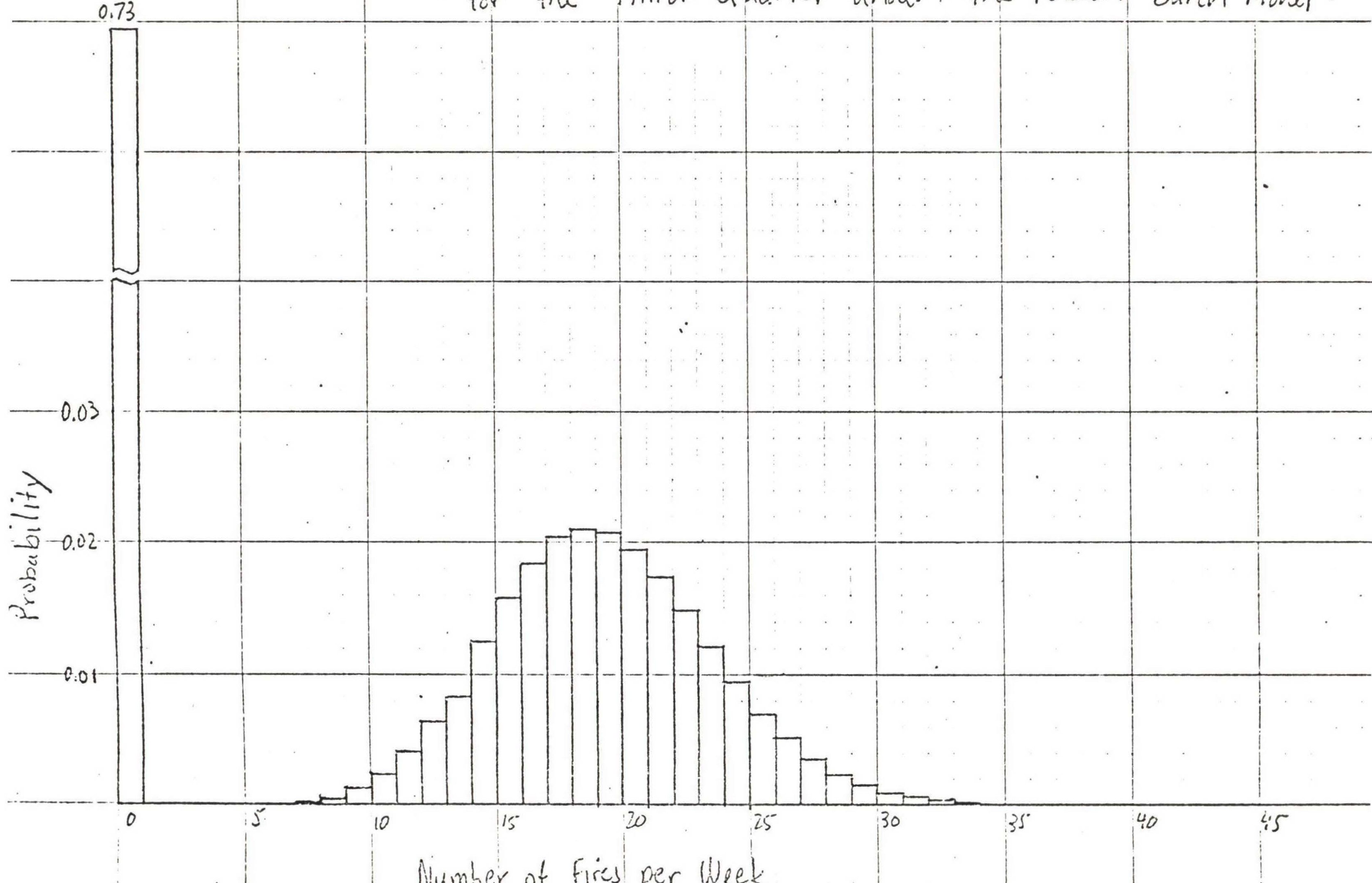


Figure 5: Theoretical Probability of Frequency of Fires per Week for the Third Quarter under the Poisson Batch model



In this case it would be assumed that a storm front brings groups of lightning storms to the forest, and that each of these lightning storms acts as a batch carrier of potential fires as described above. A probability distribution would then be assumed for the arrival of the storm fronts, and a factorial moment generating function could be derived in much the same manner as is done in Appendix (C-1). It should be noted, however, that estimation of the parameters of the probability distribution for the storm fronts will require that a corresponding number of degrees of freedom be added to the minimum degrees of freedom necessary to perform the chi-square test.

The Poisson Batch model can also be applied to fires other than lightning caused fires. As mentioned briefly above, this model could also be applied to arson caused fires. Here the batch carrier would be an arsonist who attempts to start a number of fires. The model might also apply to campers who start campfires, where each campfire started by a camper has some probability of turning into a forest fire. Unfortunately, since campers tend more to come into the forest only at certain times such as weekends and holidays, attempts to fit a probability distribution to the arrival times of campers would be difficult. This might be avoided by making the time interval over which fires are counted large enough (such as a week) to include any cyclic pattern in the arrival times of the campers. In this way the cyclic pattern in the arrival times of campers is averaged out, but only at the loss of being able to predict fires for time periods (such as during weekends) within the time interval.

There are some drawbacks in applying the Poisson Batch model, however. They include

(1) The factorial moment generating function may be hard to derive for various probability distributions.

(2) Calculating estimators by other than the method of moments may

prove difficult. This is because most other methods of estimation (such as the maximum likelihood method) require either maximization or minimization of a probability expression. Since probabilities are derived from the factorial moment generating function (which is usually quite complicated), finding such extrema will be difficult. However, in the case of the present Poisson Batch model, it does appear that the method of moment estimators for λ and μ adequately describe the distribution of the frequencies of fire occurrence per time interval.

(3) The probabilities of the frequencies may be so skewed as to prevent formation of the required minimum of cells for the chi-square test.

Despite these drawbacks, the Poisson Batch model or some adaptation of it appears to offer a way of predicting fires which depend upon a single source for ignition. Since many fires are started in this way, the model should have wide applicability.

APPENDIX (A)

THE COMPUTER PROGRAM: SELECT

```
C     COMPUTER GENERATED FORTRAN TEXT
C     PROCESSED 16-MAY-79: HMC ALTRAN V(2,7)A 2-FEB-79
CC
CC  SELECT RECORDS FROM FIRE04.DAT, AND GENERATE INTER-FIRE TIMES
CC
CC  DAVE W. SMITH, CMC '79.5   FEBRUARY, 1979
CC
CC  LAST EDIT: 9-APRIL-79
CC
CC*************************************************************
CC********  CURRENTLY SET UP FOR 15 VARIABLES  **********
CC*************************************************************
CC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCC  D A T A   D E C L A R A T I O N S   CCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
      IMPLICIT INTEGER (A-Z)
C
      DIMENSION VARNAM( 15 ), VARPNT( 15 )
      DIMENSION REC( 15 )
      EQUIVALENCE ( REC( 5 ), JDATE )
      EQUIVALENCE ( REC( 6 ), Q     )
      EQUIVALENCE ( REC( 15 ), SEQ )
      DIMENSION CLIST( 0/100 ), CTEMP( 10 )
      DIMENSION TOKLST (3/10)
      DIMENSION LINE( 80 )
C
      LOGICAL EOF, ERROR, FIND, GETVAR, SELECT
C
C  CHANNEL ASSIGNMENTS FOR I/O
C
      DATA  FIRE, DATE, Q1, Q2, Q3, Q4/
     *          1,   20, 21, 22, 23, 24/
C
C  VARIABLE NAMES, IN ORDER
C
      DATA VARNUM / 15 /
      DATA VARNAM /
     *  '*ERR',
     *  'F',
     *  '*DATE',
     *  '*TIME',
     *  '*JDAT',
     *  'Q',
     *  'SPC',
     *  'CAUSE',
     *  'SIZE',
     *  'FT',
     *  'FG',
     *  'SPF',
     *  'SFD',
     *  'FD',
     *  '*SEQ'/
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCC  M A I N   P R O G R A M  CCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
     ASSIGN 10002 TO NPR001
```

```
      GO TO 20001
10002 CONTINUE
C
      ASSIGN 10003 TO NPR002
      GO TO 20002
10003 CONTINUE
C
      ASSIGN 10004 TO NPR003
      GO TO 20003
10004 CONTINUE
C
      IF ( .NOT. (.NOT. EOF) ) GO TO 10005
10007 CONTINUE
      ASSIGN 10009 TO NPR004
      GO TO 20004
10009 CONTINUE
      IF (EOF) GO TO 10008
      ASSIGN 10010 TO NPR005
      GO TO 20005
10010 CONTINUE
      GO TO 10007
10008 CONTINUE
10005 CONTINUE
C
      ASSIGN 10011 TO NPR006
      GO TO 20006
10011 CONTINUE
C
         CALL EXIT
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCC   P R O C E D U R E S   CCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C
20001 CONTINUE
C
         OPEN ( UNIT=FIRE, DEVICE='DWS', FILE='FIRE04.DAT',
     *          ACCESS='SEQIN', BUFFER COUNT=10 )
C
         OPEN ( UNIT=DATE, DEVICE='DSK', FILE='DATE' )
C
         OPEN ( UNIT=01, FILE='01', ACCESS='SEQOUT' )
         OPEN ( UNIT=02, FILE='02', ACCESS='SEQOUT' )
         OPEN ( UNIT=03, FILE='03', ACCESS='SEQOUT' )
         OPEN ( UNIT=04, FILE='04', ACCESS='SEQOUT' )
C
         NQ1 = 0;  NQ2 = 0;  NQ3 = 0;  NQ4 = 0;
C
      GO TO 30001
C
C
20002 CONTINUE
C
C  THIS PROCEDURE PARSES SELECTION COMMANDS FROM THE USER,
C  IN THE FORM 'VARIABLE=VALUE1,VALUE2,,,VALUE12', AND BUILDS
C  A COMMAND LIST FOR THE SELECTION PROCEDURE. ARRAY USAGE IS:
C
C  VARPNT(    )  -  POINTER TO SELECTION CHUNK IN CLIST
C  CLIST( 0 )    -  POINTER TO FREE SPACE IN CLIST
```

```
C  CLIST(     )    -  SELECTION CHUNKS
C
C  SELECTION CHUNK IS N+1 WORDS LONG, WHERE (N) IS THE FIRST
   WORD OF THE CHUNK.  THE THE REMAINING N WORDS ARE POSSIBLE
C  VALUES FOR THE VARIABLE
C
C
C
      DO 10014  I =  1, VARNUM
        VARPNT( I ) = 0
10014 CONTINUE
C
        CLIST( 0 ) = 1
C
        TYPE 100
 100    FORMAT ( /' FIRE DATA SELECTOR (1,0)'/ )
C
10016 CONTINUE
        TYPE 200
 200    FORMAT ( ' > ', $)
        ACCEPT 201, LINE
 201    FORMAT ( 80A1 )
      IF ( LINE( 1 ) .EQ. ' ' ) GO TO 10017
        TOKLST( 0 ) = 0
        CPNT = 0
        LPNT = 1
        TPNT = 1
        GETVAR = .TRUE.
10018 CONTINUE
        CHAR = LINE( LPNT )
      IF ( CHAR .EQ. ' ' ) GO TO 10019
      IF ( .NOT. ( GETVAR ) ) GO TO 10020
      IF ( .NOT. ( CHAR .GE. 'A' .AND. CHAR .LE. 'Z' ) ) GO TO 10022
        CPNT = CPNT + 1
        CTEMP ( CPNT ) = CHAR
      GO TO 10023
10022 IF ( .NOT. ( CHAR .EQ. '=' ) ) GO TO 10024
        CALL PACK ( VAR, CTEMP, CPNT )
        GETVAR = .FALSE.
        CPNT = 0
10024 CONTINUE
10023 CONTINUE
      GO TO 10021
10020 CONTINUE
      IF ( .NOT. ( CHAR .EQ. ',' ) ) GO TO 10025
      ASSIGN 10027 TO NPR007
      GO TO 20007
10027 CONTINUE
      GO TO 10026
10025 CONTINUE
        CPNT = CPNT + 1
        CTEMP( CPNT ) = CHAR
10026 CONTINUE
10021 CONTINUE

        LPNT = LPNT + 1
C
      GO TO 10018
10019 CONTINUE
C
      ASSIGN 10028 TO NPR007
```

```
      GO TO 20007
10028 CONTINUE
C
        IVAR = 1
10029 CONTINUE
      IF ( VAR .EQ. VARNAM( IVAR ) .OR. IVAR .GT. VARNUM ) GO TO 10030
        IVAR = IVAR + 1
      GO TO 10029
10030 CONTINUE
C
      IF ( .NOT. ( IVAR .GT. VARNUM ) ) GO TO 10031
        TYPE 202, VAR
 202    FORMAT ( ' %NOT DEFINED - ', A5 )
      GO TO 10032
10031 CONTINUE
        VARPNT( IVAR ) = CLIST( 0 )        !LINK TO FREE LIST SPACE
        TOKPNT = 0
      DO 10033  I =  CLIST( 0 ), CLIST( 0 ) + TOKLST( 0 )
        CLIST( I ) = TOKLST( TOKPNT )
        TOKPNT = TOKPNT + 1
10033 CONTINUE
        CLIST( 0 ) = CLIST( 0 ) + TOKLST( 0 ) + 1
10032 CONTINUE
C
      GO TO 10016
10017 CONTINUE
C
        TYPE 203
 203    FORMAT (/' [PROCESSING]')
C
      GO TO 30002
CC
CC
20007 CONTINUE
C
        CALL PACK ( TOKEN, CTEMP, CPNT )
        TOKLST( 0 ) = TOKLST( 0 ) + 1
        TOKLST( TOKLST( 0 ) ) = TOKEN
        CPNT = 0
C
      GO TO 30007
CC
CC
20003 CONTINUE
C
      ASSIGN 10037 TO NPR004
      GO TO 20004
10037 CONTINUE
C
      IF ( .NOT. ( .NOT. EOF ) ) GO TO 10038
        LJDATE = REC( 5 )
        LQ     = REC( 6 )
10038 CONTINUE
C
      GO TO 30003
CC
CC
20004 CONTINUE
C
10041 CONTINUE
```

```
      ASSIGN 10043 TO NPR008
      GO TO 20008
10043 CONTINUE
      IF ( .NOT. ( .NOT. EOF ) ) GO TO 10044
      ASSIGN 10046 TO NPR009
      GO TO 20009
10046 CONTINUE
10044 CONTINUE
      IF ( SELECT .OR. EOF ) GO TO 10042
      GO TO 10041
10042 CONTINUE
C
      IF ( .NOT. ( SELECT ) ) GO TO 10047
        WRITE ( DATE, 250 ) JDATE
 250    FORMAT ( I6 )
10047 CONTINUE
C
      GO TO 30004
CC
CC
20009 CONTINUE
C
C  THIS PROCEDURE CHECKS THE SELECTION CRITERIA CHUNK IN CLIST
C  FOR EACH VARIABLE IN THE RECORD (IF A CHUNK EXISTS), UNTIL
C  THE RECORD EXPLICITLY FLUNKS OR PASSES.
C
C
        SELECT = .TRUE.
C
        IVAR = 1
C
10050 CONTINUE
      IF ( .NOT. SELECT .OR. IVAR .GT. VARNUM ) GO TO 10051
C
        CPNT = VARPNT( IVAR )
      IF ( .NOT. ( CPNT .EQ. 0 ) ) GO TO 10052
        SELECT = SELECT .AND. .TRUE.
      GO TO 10053
10052 CONTINUE
        FIND = .FALSE.
      DO 10054  I =  CPNT + 1, CPNT + CLIST( CPNT )
        FIND = FIND .OR. ( REC( IVAR ) .EQ. CLIST( I ) )
10054 CONTINUE
        SELECT = SELECT .AND. FIND
10053 CONTINUE
        IVAR = IVAR + 1
      GO TO 10050
10051 CONTINUE
C
      GO TO 30009
CC
CC
20008 CONTINUE
C
        READ ( FIRE, 500, END=10057) REC
      EOF  = .FALSE.
      GO TO 10058
10057 EOF  = .TRUE.
10058 CONTINUE
 500    FORMAT ( A1,A2,A4,A2,I6,A1,A2,4A1,2A2,A1,9X,A4 )
```

```
C
      GO TO 30008
CC
CC
20005 CONTINUE
C
        GAP = JDATE - LJDATE
C
      IF ( .NOT. ( LQ .EQ. '1' ) ) GO TO 10060
        NQ1 = NQ1 + 1
        WRITE ( Q1, 600 ) GAP
      GO TO 10061
10060 IF ( .NOT. ( LQ .EQ. '2' ) ) GO TO 10062
        NQ2 = NQ2 + 1
        WRITE ( Q2, 600 ) GAP
      GO TO 10061
10062 IF ( .NOT. ( LQ .EQ. '3' ) ) GO TO 10063
        NQ3 = NQ3 + 1
        WRITE ( Q3, 600 ) GAP
      GO TO 10061
10063 IF ( .NOT. ( LQ .EQ. '4' ) ) GO TO 10064
        NQ4 = NQ4 + 1
        WRITE ( Q4, 600 ) GAP
10064 CONTINUE
10061 CONTINUE
C
 600    FORMAT ( 1X,I6)
C
        LJDATE = JDATE
        LQ     = Q
C
      GO TO 30005
CC
CC
20006 CONTINUE
C
        CLOSE ( UNIT = FIRE )
        CLOSE ( UNIT = DATE )
        CLOSE ( UNIT = Q1 )
        CLOSE ( UNIT = Q2 )
        CLOSE ( UNIT = Q3 )
        CLOSE ( UNIT = Q4 )
C
        TYPE 700, NQ1, NQ2, NQ3, NQ4
 700    FORMAT (/' TOTAL OF ',4(I4,','),' FIRES SELECTED')
C
      GO TO 30006
CC
CC
30001 GO TO NPR001, (10002)
30002 GO TO NPR002, (10003)
30003 GO TO NPR003, (10004)
30004 GO TO NPR004, (10009,10037)
30005 GO TO NPR005, (10010)
30006 GO TO NPR006, (10011)
30007 GO TO NPR007, (10027,10028)
30008 GO TO NPR008, (10043)
30009 GO TO NPR009, (10046)
      STOP
      END
```

```
/TRACE 0,
/LINE UP
          PROGRAM SELECT
CC
CC  SELECT RECORDS FROM FIRE04.DAT, AND GENERATE INTER-FIRE TIMES
CC
CC  DAVE W. SMITH, CMC '79.5   FEBRUARY, 1979
CC
CC  LAST EDIT: 9-APRIL-79
CC
CC******************************************************
CC*********  CURRENTLY SET UP FOR 15 VARIABLES  **********
CC******************************************************
CC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCC  D A T A   D E C L A R A T I O N S  CCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
          :
          :     IMPLICIT INTEGER (A-Z)
          :
          :     DIMENSION VARNAM( 15 ), VARPNT( 15 )
          :     DIMENSION REC( 15 )
          :     EQUIVALENCE ( REC( 5 ), JDATE )
          :     EQUIVALENCE ( REC( 6 ), Q     )
          :     EQUIVALENCE ( REC( 15 ), SEQ )
          :     DIMENSION CLIST( 0/100 ), CTEMP( 10 )
          :     DIMENSION TOKLST (0/10)
          :     DIMENSION LINE( 80 )
          :
          :     LOGICAL EOF, ERROR, FIND, GETVAR, SELECT
C         :
C   CHANNEL ASSIGNMENTS FOR I/O
C         :
          :     DATA  FIRE, DATE, Q1, Q2, Q3, Q4/
         *:            1,   20, 21, 22, 23, 24/
C         :
C   VARIABLE NAMES, IN ORDER
C         :
          :     DATA VARNUM / 15 /
          :     DATA VARNAM /
         *:     '*ERR',
         *:     'F',
         *:     '*DATE',
         *:     '*TIME',
         *:     '*JDAT',
         *:     'Q',
         *:     'SPC',
         *:     'CAUSE',
         *:     'SIZE',
         *:     'FT',
         *:     'FG',
         *:     'SPF',
         *:     'SFD',
         *:     'FD',
         *:     '*SEQ'/
C         :
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
CCCCCCCCCCCCCCC   M A I N   P R O G R A M   CCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
             :
             :     PERFORM ( OPEN FILES AND RESET COUNTERS )
             :
             :     PERFORM ( PARSE SELECTION CRITERIA )
             :
             :     PERFORM ( FIND FIRST RECORD )
             :
             :     IF (.NOT. EOF) THEN
             :  :     REPEAT
             :  :  :     PERFORM ( FIND NEXT RECORD )
             :  :     UNTIL (EOF)
             :  :  :     PERFORM ( GENERATE INTER-FIRE TIMES )
             :  :     END REPEAT
             :     END IF
             :
             :     PERFORM ( CLOSE FILES AND DISPLAY COUNTERS )
             :
             :     CALL EXIT
             :
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCC   P R O C E D U R E S   CCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C            :
C            :
             :     PROCEDURE ( OPEN FILES AND RESET COUNTERS )
             :  :
             :  :     OPEN ( UNIT=FIRE, DEVICE='DWS', FILE='FIRE24.DAT',
            *:  :             ACCESS='SEQIN', BUFFER COUNT=10 )
             :  :
             :  :     OPEN ( UNIT=DATE, DEVICE='DSK', FILE='DATE' )
             :  :
             :  :     OPEN ( UNIT=Q1, FILE='Q1', ACCESS='SEQOUT' )
             :  :     OPEN ( UNIT=Q2, FILE='Q2', ACCESS='SEQOUT' )
             :  :     OPEN ( UNIT=Q3, FILE='Q3', ACCESS='SEQOUT' )
             :  :     OPEN ( UNIT=Q4, FILE='Q4', ACCESS='SEQOUT' )
             :  :
             :  :     NQ1 = 0;  NQ2 = 0;  NQ3 = 0;  NQ4 = 0;
             :  :
             :     END PROCEDURE
C            :
C            :
             :     PROCEDURE ( PARSE SELECTION CRITERIA )
C            :  :
C  THIS PROCEDURE PARSES SELECTION COMMANDS FROM THE USER,
C  IN THE FORM 'VARIABLE=VALUE1,VALUE2,,VALUE12', AND BUILDS
C  A COMMAND LIST FOR THE SELECTION PROCEDURE. ARRAY USAGE IS:
C            :  :
C  VARPNT(   )  -  POINTER TO SELECTION CHUNK IN CLIST
C  CLIST( 0 )   -  POINTER TO FREE SPACE IN CLIST
C  CLIST(   )   -  SELECTION CHUNKS
C            :  :
C  SELECTION CHUNK IS N+1 WORDS LONG, WHERE (N) IS THE FIRST
C  WORD OF THE CHUNK.  THE THE REMAINING N WORDS ARE POSSIBLE
C  VALUES FOR THE VARIABLE
C            :  :
             :  :
             :  :     DO I: 1, VARNUM
             :  :  :     VARPNT( I ) = 0
```

```
          :  :  END DO
          :  :
          :  :  CLIST( Ø ) = 1
          :  :
          :  :  TYPE 100
  100     :  :  FORMAT ( /' FIRE DATA SELECTOR (1.0)'/ )
          :  :
          :  :  REPEAT
          :  :  :  TYPE 200
  200     :  :  :  FORMAT ( ' > ', $)
          :  :  :  ACCEPT 201, LINE
  201     :  :  :  FORMAT ( 80A1 )
          :  :  UNTIL ( LINE( 1 ) .EQ. ' ' )
          :  :  :  TOKLST( Ø ) = Ø
          :  :  :  CPNT = 0
          :  :  :  LPNT = 1
          :  :  :  TPNT = 1
          :  :  :  GETVAR = .TRUE.
          :  :  :  REPEAT
          :  :  :  :  CHAR = LINE( LPNT )
          :  :  :  :  TRACE (5, CHAR: A1 )
          :  :  :  UNTIL ( CHAR .EQ. ' ' )
          :  :  :  :  IF ( GETVAR ) THEN
          :  :  :  :  :  IF ( CHAR .GE. 'A' .AND. CHAR .LE. 'Z' ) THEN
          :  :  :  :  :  :  CPNT = CPNT + 1
          :  :  :  :  :  :  CTEMP ( CPNT ) = CHAR
          :  :  :  :  :  :  TRACE ( 4, CPNT,CHAR: (I1,1X,A1) )
          :  :  :  :  :  ELSE IF ( CHAR .EQ. '=' ) THEN
          :  :  :  :  :  :  CALL PACK ( VAR, CTEMP, CPNT )
          :  :  :  :  :  :  TRACE (3, VAR,CPNT: (A5,I2) )
          :  :  :  :  :  :  GETVAR = .FALSE.
          :  :  :  :  :  :  CPNT = Ø
          :  :  :  :  :  END IF
          :  :  :  :  ELSE
          :  :  :  :  :  IF ( CHAR .EQ. ',' ) THEN
          :  :  :  :  :  :  PERFORM ( PACK AND STORE TOKEN )
          :  :  :  :  :  ELSE
          :  :  :  :  :  :  CPNT = CPNT + 1
          :  :  :  :  :  :  CTEMP( CPNT ) = CHAR
          :  :  :  :  :  END IF
          :  :  :  :  END IF
          :  :  :  :
          :  :  :  :  LPNT = LPNT + 1
          :  :  :  :
          :  :  :  END REPEAT
          :  :  :
          :  :  :  PERFORM ( PACK AND STORE TOKEN )
          :  :  :
          :  :  :  IVAR = 1
          :  :  :  REPEAT
          :  :  :  UNTIL ( VAR .EQ. VARNAM( IVAR ) .OR. IVAR .GT. VARNUM )
          :  :  :  :  IVAR = IVAR + 1
          :  :  :  END REPEAT
          :  :  :
          :  :  :  IF ( IVAR .GT. VARNUM ) THEN
          :  :  :  :  TYPE 202, VAR
  202     :  :  :  :  FORMAT ( ' %NOT DEFINED - ', A5 )
          :  :  :  ELSE
          :  :  :  :  TRACE (2, CLIST(Ø): I3 )
          :  :  :  :  VARPNT( IVAR ) = CLIST( Ø )      !LINK TO FREE LIST SPACE
```

```
          :  :  :  :     TOKPNT = Ø
          :  :  :  :     TRACE (2, TOKLST(Ø): I2 )
          :  :  :  :     DO I: CLIST( Ø ), CLIST( Ø ) + TOKLST( Ø )
          :  :  :  :  :     CLIST( I ) = TOKLST( TOKPNT )
          :  :  :  :  :     TRACE (2, I,CLIST(I): I3,0 )
          :  :  :  :  :     TOKPNT = TOKPNT + 1
          :  :  :  :     END DO
          :  :  :  :     CLIST( Ø ) = CLIST( Ø ) + TOKLST( Ø ) + 1
          :  :  :     END IF
          :  :  :
          :  :     END REPEAT
          :  :
          :  :     TYPE 203
     203  :  :     FORMAT (/' [PROCESSING]')
          :  :
          :     END PROCEDURE
  CC
  CC
          :     PROCEDURE ( PACK AND STORE TOKEN )
          :  :
          :  :     CALL PACK ( TOKEN, CTEMP, CPNT )
          :  :     TRACE ( 3, TOKEN,CPNT: (A5,I2) )
          :  :     TOKLST( Ø ) = TOKLST( Ø ) + 1
          :  :     TOKLST( TOKLST( Ø ) ) = TOKEN
          :  :     CPNT = Ø
          :  :
          :     END PROCEDURE
  CC
  CC
          :     PROCEDURE ( FIND FIRST RECORD )
          :  :
          :  :     PERFORM ( FIND NEXT RECORD )
          :  :
          :  :     IF ( .NOT. EOF ) THEN
          :  :  :     LJDATE = REC( 5 )
          :  :  :     LQ     = REC( 6 )
          :  :     END IF
          :  :
          :     END PROCEDURE
  CC
  CC
          :     PROCEDURE ( FIND NEXT RECORD )
          :  :
          :  :     REPEAT
          :  :  :     PERFORM ( READ NEXT RECORD )
          :  :  :     IF ( .NOT. EOF ) THEN
          :  :  :  :     PERFORM ( TEST SELECTION CRITERIA )
          :  :  :     END IF
          :  :     UNTIL ( SELECT .OR. EOF )
          :  :     END REPEAT
          :  :
          :  :     IF ( SELECT ) THEN
          :  :  :     WRITE ( DATE, 250 ) JDATE
     250  :  :  :     FORMAT ( I6 )
          :  :  :     TRACE ( 1, SEQ: ('ACCEPT #',A5) )
          :  :     END IF
          :  :
          :     END PROCEDURE
  CC
  CC
```

```
          !     PROCEDURE ( TEST SELECTION CRITERIA )
C         :  !
C   THIS PROCEDURE CHECKS THE SELECTION CRITERIA CHUNK IN CLIST
C   FOR EACH VARIABLE IN THE RECORD (IF A CHUNK EXISTS), UNTIL
C   THE RECORD EXPLICITLY FLUNKS OR PASSES.
C         :  :
          :  :
          :  :     SELECT = .TRUE.
          :  :
          :  :     IVAR = 1
          :  :
          :  :     REPEAT
          :  :     UNTIL ( .NOT. SELECT .OR. IVAR .GT. VARNUM )
          :  :  !
          :  :  !     CPNT = VARPNT( IVAR )
          :  :  !     TRACE ( 2, IVAR,VARPNT(IVAR): 2I4 )
          :  :  !     IF ( CPNT .EQ. 0 ) THEN
          :  :  !  :     SELECT = SELECT .AND. .TRUE.
          :  :  !     ELSE
          :  :  !  :     FIND = .FALSE.
          :  :  !  :     TRACE ( 3, CPNT,CLIST( CPNT ): 2I4 )
          :  :  !  :     DO I: CPNT + 1, CPNT + CLIST( CPNT )
          :  :  !  :  :     TRACE ( 2, REC(IVAR),CLIST(I): ('-'A5'-'A5'-') )
          :  :  !  :  :     FIND = FIND .OR. ( REC( IVAR ) .EQ. CLIST( I ) )
          :  :  !  :     END DO
          :  :  !  :     SELECT = SELECT .AND. FIND
          :  :  !     END IF
          :  :  !     IVAR = IVAR + 1
          :  :     END REPEAT
          :  :
          :     END PROCEDURE
CC
CC
          :     PROCEDURE ( READ NEXT RECORD )
          :  :
          :  :     READ ( FIRE, 500, END:EOF ) REC
  500     :  :     FORMAT ( A1,A2,A4,A2,I6,A1,A2,4A1,2A2,A1,9X,A4 )
          :  :     TRACE ( 1, SEQ: ('READ #',A5) )
          :  :
          :     END PROCEDURE
CC
CC
          :     PROCEDURE ( GENERATE INTER-FIRE TIMES )
          :  :
          :  :     GAP = JDATE - LJDATE
          :  :
          :  :     IF      ( LQ .EQ. '1' ) THEN
          :  :  !     NQ1 = NQ1 + 1
          :  :  !     WRITE ( Q1, 600 ) GAP
          :  :     ELSE IF ( LQ .EQ. '2' ) THEN
          :  :  !     NQ2 = NQ2 + 1
          :  :  !     WRITE ( Q2, 600 ) GAP
          :  :     ELSE IF ( LQ .EQ. '3' ) THEN
          :  :  !     NQ3 = NQ3 + 1
          :  :  !     WRITE ( Q3, 600 ) GAP
          :  :     ELSE IF ( LQ .EQ. '4' ) THEN
          :  :  !     NQ4 = NQ4 + 1
          :  :  !     WRITE ( Q4, 600 ) GAP
          :  :     END IF
          :  :
```

APPENDIX (B)

APPLICATION OF GOODNESS-OF-FIT TESTS
TO THE EXPONENTIAL DISTRIBUTION FUNCTION

```
  600   :  :      FORMAT ( 1X,I6)
        :  :
        :  :      LJDATE = JDATE
        :  :      LQ     = Q
        :  :
        :      END PROCEDURE
 CC
 CC
        :      PROCEDURE ( CLOSE FILES AND DISPLAY COUNTERS )
        :  :
        :  :      CLOSE ( UNIT = FIRE )
        :  :      CLOSE ( UNIT = DATE )
        :  :      CLOSE ( UNIT = Q1 )
        :  :      CLOSE ( UNIT = Q2 )
        :  :      CLOSE ( UNIT = Q3 )
        :  :      CLOSE ( UNIT = Q4 )
        :  :
        :  :      TYPE 700, NQ1, NQ2, NQ3, NQ4
  700   :  :      FORMAT (/' TOTAL OF ',4(I4,','),' FIRES SELECTED')
        :  :
        :      END PROCEDURE
 CC
 CC
            END
```

```
      AVE= SUM/N
      STAT= 0.
      SUP= 0.
C
C COMPUTING THE STATISTIC
C
      TYPE 60
60    FORMAT(/5X,'TIME',T23,'DELTA'/)
      DO 90 J= 1,N
      READ(1,40) K
      TIME= K
      I= J
      RES= DELTA(TIME,AVE,I,N)
      STAT= STAT+RES
      TYPE 70, K, RES
70    FORMAT(1X,I8,T22,F6.4)
      IF (RES-SUP) 90,90,80
80    SUP= RES
90    CONTINUE
      TYPE 100, N, AVE, SUP, STAT
100   FORMAT(/1X,'N = ',I3//1X,'MEAN = ',F7.2//1X,
     1  'MAX DELTA = ',F6.4//1X,'STATISTIC = ',F9.4)
      END
C
C THE FOLLOWING FUNCTION COMPUTES THE DELTAS
C
C IMPORTANT VARIABLES
C
C DIST     VALUE OF EXPONENTIAL DISTRIBUTION
C          (PARAMETER IS ESTIMATED BY THE MEAN
C           OF THE INTER-FIRE TIMES)
C
C UPPER    DIFFERENCE BETWEEN THE EMPIRICAL AND
C          THEORETICAL DISTRIBUTIONS EVALUATED
C          AT THE RIGHT END OF THE INTERVAL
C
C ALOWER   DIFFERENCE BETWEEN THE EMPIRICAL AND
C          THEORETICAL DISTRIBUTIONS EVALUTED
C          AT THE LEFT END OF THE INTERVAL
C
      REAL FUNCTION DELTA(TIME,AVE,A,N)
      DIST= 1-EXP((-TIME)/AVE)
      UPPER= ABS(A/N-DIST)
      ALOWER= ABS(DIST-(A-1)/N)
      IF (ALOWER-UPPER) 10,10,20
10    DELTA= UPPER
      GOTO 30
20    DELTA= ALOWER
30    END
```

```
C
C PROGRAM EXPFIT
C
C AUTHOR: MILTON SCRITSMIER
C
C DATE WRITTEN: MARCH, 1979
C
C
C THIS PROGRAM TESTS DATA OBTAINED FROM THE PROGRAM
C SELECT FOR AN EXPONENTIAL FIT BY USING BOTH
C THE STANDARD K-S TEST AND THE MODIFIED K-S TEST.
C
C INPUT REQUIRES ONE OF THE QN.DAT FILES OBTAINED
C FROM THE SELECT PROGRAM WHICH CONTAINS THE
C THE ORDERED INTER-FIRE TIMES STARTING FROM
C THE BEGINNING OF THE QUARTER (ORDERING MAY
C BE ACCOMPLISHED FOR ALL THE QN.DAT FILES BY TYPING
C
C            .DO QSORT
C
C TO THE MONITOR DOT AFTER RUNNING THE SELECT
C PROGRAM).  OUTPUT IS THE STANDARD K-S STATISTIC
C ('MAX DELTA') AND THE MODIFIED K-S STATISTIC
C ('STATISTIC').
C
C IMPORTANT VARIABLES
C
C NUM   TIME OF FIRE FROM BEGINNING OF THE QUARTER
C
C N     TOTAL NUMBER OF FIRES OBSERVED IN THE QUARTER
C
C SUM   SUM OF THE FIRE TIMES
C
C TIME  TIME OF FIRE FROM THE BEGINNING OF THE
C       QUARTER (REAL VARIABLE TYPE FOR COMPUTATIONAL
C       ACCURACY)
C
C SUM   STANDARD K-S STATISTIC
C
C STAT  MODIFIED K-S STATISTIC
C
C
C COUNTING AND SUMMING THE ELEMENTS IN THE FILE
C
      REAL I
      TYPE 10
10    FORMAT(1X,32HENTER THE QUARTER OF THE YEAR AS
     *       19H 'QN', N = 1,2,3,4.)
      ACCEPT 20, QUART
20    FORMAT(A2)
      N= 0
      J= 0
      OPEN(UNIT= 1,FILE= QUART)
30    READ(1,40,END= 50) NUM
40    FORMAT(I7)
      N= N+1
      J= J+NUM
      GOTO 30
50    REWIND 1
      SUM= J
```

EXPONENTIAL FIT FOR FIRES BY
CAUSE AND FUEL TYPE

I. Cause = Other (Man-Made)

First Quarter

Fuel Type	Number of Fires	Max Delta $(\hat{D}_n)$	Sum of Deltas (S^*_n)	Result	Mean
A	55	0.0831	2.2065	Accept at $\alpha \geq 0.20$	237.80
B	42	0.0860	1.7339	Accept at $\alpha \geq 0.20$	402.10
C	3	0.4534	0.8728	Accept at $\alpha \geq 0.20$	1893.67
F	4	0.2631	0.8851	Accept at $\alpha \geq 0.20$	1233.25
G	39	0.1950	3.3456		353.08
H	4	0.3865	1.1219	Accept at $\alpha \geq 0.10$	1870.75
I	1				
J	0				
K	11	0.2794	1.6165	Accept at $\alpha \geq 0.10$	810.36
L	6	0.3702	1.3068	Accept at $\alpha \geq 0.10$	1902.17
T	20	0.1491	1.6319	Accept at $\alpha \geq 0.20$	574.30
U	0				

Second Quarter

Fuel Type	Number of Fires	Max Delta $(\hat{D}_n)$	Sum of Deltas (S^*_n)	Result	Mean
A	600	0.1592	49.1949		35.68
B	222	0.1689	16.2852		88.28
C	27	0.1052	1.2611	Accept at 0.20	641.78
F	50	0.1748	3.3745		465.18
G	258	0.1340	15.6446		76.94
H	23	0.2786	3.9698	Reject at = 0.01	1333.57
I	1				
J	0				
K	44	0.0978	2.0072	Accept at 0.20	390.11
L	15	0.1824	1.2948	Accept at 0.20	1499.53
T	178	0.1028	7.0107		117.01
U	1				

Third Quarter

Fuel Type	Number of Fires	Max Delta ($\hat{D}_n$)	Sum of Deltas (S^*_n)	Result	Mean
A	577	0.1550	46.5031		43.99
B	285	0.1820	28.3439		93.07
C	20	0.1911	1.4467	Accept at $\alpha \geq 0.20$	1431.90
F	47	0.3252	8.6277	Reject at $\alpha = 0.01$	763.91
G	244	0.1864	25.7452		122.73
H	28	0.2298	3.7615	Reject at $\alpha = 0.01$	932.43
I	0				
J	0				
K	45	0.2385	5.8319		780.96
L	19	0.1435	1.3370	Accept at $\alpha \geq 0.20$	1348.42
T	175	0.1363	13.2667		161.09
U	6	0.1969	0.8589	Accept at $\alpha \geq 0.20$	8709.17

Fourth Quarter

Fuel Type	Number of Fires	Max Delta ($\hat{D}_n$)	Sum of Deltas (S^*_n)	Result	Mean
A	34	0.3334	6.2787	Reject at $\alpha = 0.01$	297.44
B	18	0.2570	2.2099	Accept at $\alpha = 0.01$	307.33
C	6	0.2990	1.1202	Accept at $\alpha \geq 0.20$	2130.33
F	1	0.6321	0.6321		
G	11	0.1468	1.0767	Accept at $\alpha \geq 0.20$	485.00
H	1	0.6321	0.6321		
I	0				
J	0				
K	9	0.3522	1.4825	Accept at $\alpha \geq 0.10$	685.00
L	3	0.3631	0.8118	Accept at $\alpha \geq 0.20$	2590.33
T	11	0.3455	1.9113	Accept at $\alpha = 0.01$	775.27
U	0				

APPENDIX (C-1)

DERIVATION OF THE FACTORIAL MOMENT GENERATING FUNCTION FOR THE POISSON BATCH MODEL

Let M be the random variable representing the total number of lightning storms arriving at the forest per time interval (such as a week or a month). As was mentioned in the main body of the report, it is assumed that M has a Poisson distribution with rate parameter λ. Next let Y_j be the random variable representing the total number of fires started by the j^{th} lightning storm during the time interval. It is also assumed that Y_j has a Poisson distribution, this time with rate parameter μ. If N is the random variable representing the total number of lightning caused fires occuring during the time interval, then

$$N = \sum_{j=0}^{M} Y_j \quad \text{where} \quad Y_0 \equiv 0$$

We are interested in the factorial moment generating function $\psi_N(t)$ for N, which is defined by

$$\psi_N(t) = E[t^N]$$

Since $N = Y_0 + Y_1 + \ldots + Y_M$, we have

$$\psi_N(t) = E[t^{Y_0+Y_2+\ldots+Y_M}]$$

Because of this last expression we can derive $\psi_N(t)$ by conditioning on M as follows:

$$E[t^{Y_0+Y_1+\ldots+Y_M}] = \sum_{m=0}^{\infty} P[M=m]\, E[t^{Y_0+Y_1+\ldots+Y_M} | M=m]$$

Now

$$E[t^{Y_0+Y_1+\ldots+Y_M} | M=m] = E[t^{Y_0+Y_1+\ldots+Y_m}]$$

If we now assume that the Y_j are independent

$$E[t^{Y_0+Y_1+\ldots+Y_m}] = E[t^{Y_0}]\; E[t^{Y_1}] \;\ldots\; E[t^{Y_m}]$$

Since all the Y_j except Y_0 are assumed to have the same Poisson distribution with parameter μ , this becomes

$$E[t^{Y_0}]\; E[t^{Y_1}] \;\ldots\; E[t^{Y_m}] = E[t^{Y_0}]\; (E[t^{Y_1}])^m$$

Since $Y_0 \equiv 0$, $E[t^{Y_0}] = 1$. Also, for Y_1 with a Poisson distribution we have

$$E[t^{Y_1}] = \sum_{y=0}^{\infty} t^y\, P[Y_1=y]$$

$$= \sum_{y=0}^{\infty} t^y e^{-\mu} \frac{\mu^Y}{y!}$$

$$= e^{-\mu} \sum_{y=0}^{\infty} \frac{(\mu t)^Y}{y!}$$

The sum is just the Taylor's series for $e^{\mu t}$. Thus

$$E[t^{Y_1}] = e^{-\mu(1-t)}$$

Hence

$$E[t^{Y_0+Y_1+\ldots+Y_m} | M=m] = (e^{-\mu(1-t)})^m$$

which is true even in the case $m = 0$. Then

$$E[t^N] = \sum_{m=0}^{\infty} P[M=m]\; (e^{-\mu(1-t)})^m$$

Since M has a Poisson distribution with rate parameter λ,

$$P[M=m] = \frac{\lambda^m}{m!} e^{-\lambda}$$

so

$$E[t^N] = \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} e^{-\lambda} (e^{-\mu(1-t)})^m$$

$$= e^{-\lambda} \sum_{m=0}^{\infty} \frac{(\lambda e^{-\mu(1-t)})^m}{m!}$$

This last sum is just the Taylor's series for $\exp(\lambda e^{-\mu(1-t)})$. Thus

$$\psi_N(t) = E[t^N] = e^{-\lambda} e^{b(t)}$$

$$\psi_N(t) = e^{-\lambda+b(t)} \quad \text{where} \quad b(t) = \lambda e^{-\mu(1-t)} \tag{1}$$

In order to use the factorial moment generating function to estimate the parameters λ and μ and to find probabilities, we must be able to find the n^{th} derivative of $\psi_N(t)$. It is shown below that

$$\psi_N^{(n)}(t) = \mu^n \psi_N(t) \sum_{j=1}^{n} a_{j,n} (b(t))^j \tag{2}$$

where

$$a_{1,1} = a_{1,2} = a_{2,2} = 1$$

and for $n = 3, 4, 5, \ldots$ the coefficients are defined recursively by

$$a_{1,n} = 1$$

$$a_{i,n} = ai_{i,n-1} + a_{i-1,n-1} \qquad i = 2, 3, 4, \ldots, n-1$$

$$a_{n,n} = 1$$

Expression (2) can be proved by induction on n. For $n=1$, we have

$$\psi_N^{(1)}(t) = \frac{d}{dt}\,(e^{-\lambda+b(t)})$$

$$= (\frac{d}{dt}\,b(t))\,(e^{-\lambda+b(t)})$$

$$= (\frac{d}{dt}\,(\lambda e^{-\mu(1-t)}))\,\psi_N(t)$$

$$= \mu\lambda e^{-\mu(1-t)}\,\psi_N(t)$$

$$\psi_N^{(1)}(t) = \mu b(t)\,\psi_N(t)$$

For $n = 2$ we have

$$\psi_N^{(2)}(t) = \mu b^{(1)}(t)\,\psi_N(t) + \mu b(t)\,\psi_N^{(1)}(t)$$

$$= \mu(\mu b(t))\,\psi_N(t) + \mu b(t)(\mu b(t)\,\psi_N(t))$$

$$\psi_N^{(2)}(t) = \mu^2\,\psi_N(t)(b(t) + (b(t))^2)$$

Now assume that expression (2) is true for $n - 1$. Then

$$\psi_N^{(n-1)}(t) = \mu^{n-1}\,\psi_N(t)\sum_{j=1}^{n-1} a_{j,n-1}(b(t))^j$$

and from this

$$\psi_N^{(n)}(t) = \mu^{n-1}\,\{\psi_N^{(1)}(t)\sum_{j=1}^{n-1}(a_{j,n-1}(b(t))^j)$$

$$+ \psi_N(t)\sum_{j=1}^{n-1}(ja_{j,n-1}\,b^{(1)}(t)(b(t))^{j-1})\}$$

$$= \mu^{n-1}\,\{\mu b(t)\,\psi_N(t)\sum_{j=1}^{n-1}(a_{j,n-1}(b(t))^j)$$

$$+ \psi_N(t)\sum_{j=1}^{n-1}(ja_{j,n-1}\mu b(t)(b(t))^{j-1})\}$$

$$\psi_N^{(n)}(t) = \mu^n \psi_N(t) \{ \sum_{j=1}^{n-1} (a_{j,n-1}(b(t))^{j+1}) + \sum_{j=1}^{n-1} (ja_{j,n-1}(b(t)^j)\}$$

$$= \mu^n \psi_N(t)\{a_{1,n-1}b(t) + \sum_{j=2}^{n} (a_{j-1,n-1}, (b(t))^j)$$

$$+ \sum_{j=2}^{n-1} (ja_{j,n-1}(b(t))^j)\}$$

For each power j of $b(t)$ we have (remembering that a_{jn} is the coefficient of the j^{th} power of $b(t)$).

$$j = 1: \quad a_{1,n} = a_{1,n-1} = 1$$

$$j = 2, 3, \ldots, n-1: \quad a_{jn} = a_{j-1,n-1} + ja_{j,n-1}$$

$$j = n: \quad a_{n,n} = a_{n-1,n-1} = 1$$

Thus expression (2) is true for n also. Then by induction expression (2) is true for all n.

To estimate the parameters λ and μ, we note that the factorial mement generating function has the property that

$$E[N(N-1) \cdots (N-n+1)] = \psi_N^{(n)}(1)$$

$$= \mu^n \sum_{j=1}^{n} a_{j,n}\lambda^j \qquad \text{(by (2))}$$

In particular,

$$E[N] = \mu\lambda \tag{3}$$

and

$$E[N(N-1)] = \mu^2(\lambda+\lambda^2)$$

Since

$$E[N(N-1)] = E[N^2-N]$$

$$= E[N^2] - E[N]$$

we see that

$$E[N^2] = E[N(N-1)] + E[N]$$

$$= \mu^2(\lambda+\lambda^2) + \mu\lambda$$

$$E[N^2] = \mu\lambda(1+\mu+\mu\lambda) \qquad (4)$$

If N is the total number of fires occurring during a week, it was shown in the main body of the report that for a given quarter over the eight years of available data we will have a random sample consisting of 104 observations of N (note that it is implicitly assumed here that λ and μ are constant over the quarter). If we compute the first and second moments of this random sample, (call them e_1 and e_2 respectively), by the method of moments we set

$$e_1 = E[N]$$

$$e_2 = E[N^2] \quad .$$

Thus from expressions (3) and (4) we get the following set of simultaneous equations for the estimated parameters $\hat{\lambda}$ and $\hat{\mu}$:

$$e_1 = \hat{\lambda}\hat{\mu}$$

$$e_2 = \hat{\lambda}\hat{\mu}(1+\hat{\mu} + \hat{\mu}\hat{\lambda})$$

Solving for $\hat{\lambda}$ and $\hat{\mu}$ in terms of e_1 and e_2 gives

$$\hat{\mu} = \frac{e_2}{e_1} - e - 1$$

$$\hat{\lambda} = \frac{e_1}{\hat{\mu}}$$

These expressions are the method of moments estimates for μ and λ.

Another property of the factorial moment generating function is that

$$P[N=n] = \frac{1}{n!}\,\psi_N^{(n)}(0) \qquad n = 0, 1, 2, \ldots$$

so

$$P[N=n] = \frac{\mu^n}{n!}\, e^{-\lambda(1-e^{-\mu})} \sum_{j=1}^{n} a_{j,n} (\lambda e^{-\mu})^j \qquad \text{for} \qquad n = 1, 2, \ldots$$

and

$$P[N=0] = e^{-\lambda(1-e^{-\mu})}$$

The calculation of $P[N=n]$ is necessary to perform the chi-square test, as is seen in Appendix (C-2).

APPENDIX (C -2)

THE PROGRAM POSSON AND THE CHI-SQUARE TEST

The computer program POSSON (which is listed in Appendix (C -3) is the means by which the fire occurrence frequency per time interval is determined, the means by which the parameters are estimated, and the means by which the chi-square goodness of fit test is performed. The program, although listed in FORTRAN in Appendix (C -3), was originally written in ALTRAN a preprocessor for FORTRAN which is available at Harvey Mudd College. ALTRAN allows ALGOL-like construction of FORTRAN programs. As a result, programs are more readable, and decision blocks can be more simply expressed. For this reason (and the fact that the FORTRAN program produced by the ALTRAN preprocessor may not always contain the most natural expression of algorithms), it is recommended that any major changes on POSSON done at Harvey Mudd College be done in ALTRAN.

The program POSSON is divided into several subprograms, each of which is listed here.

Main Program: The main program contains the algorithm for the formation of cells for the chi-square test and calculates the chi-square statistic if a sufficient number of cells can be formed for the test.

Subroutine FRQNCY: The subroutine FRQNCY asks for the quarter of the year for which the test is to be performed. For the specified quarter of each year for which there is data, the subroutine generates the first time interval within the quarter, and counts the number of lightning fires that occurred within this time interval. This process is repeated for each time interval within the quarter. The final result of this process is a record of the number of time intervals for all

the quarters for which a given frequency of fires occurred.

Subroutine PAREST: This subroutine estimates the parameters λ and μ of the Poisson Batch model by the method of moments as outlined in Appendix (C-1). Execution of this subroutine requires previous execution of the subroutine FRQNCY to obtain the frequencies of fire occurrence.

Function MOMGEN: This function calculates the moment generating function and its derivatives from the estimated parameters for use in calculating probabilities. The calculations needed for this are given in Appendix (C-1).

Function PROB: This function calculates the probability of a given frequency of fires per time interval from the moment generating function as derived at the end of Appendix (C-1). A method of calculation using logarithms is used when the calculation of the factorial function exceeds the bounds of the system.

BLOCK DATA Subprogram: This Subprogram performs the required initialization of the data in COMMON. Included among these variables are variables which give the length of the time interval, the minimum expectation required to form a chi-square cell, and which also give values for certain computational limits of the computer. These variables are listed and described at the beginning of the BLOCK DATA subprogram.

A few words about how the first and second moments of the data are calculated, and about how the cells for the chi-square test are formed must be given.

The random sample of data upon which the Poisson Batch model is based is the number N_i of fires that occurred during the i^{th} time interval. Thus

to estimate λ and μ we need the first and second moments of the Ni. The subroutine FRQNCY, however, provides only the number F_j of time intervals which had j fires in them, i.e., the number F_j of the N_i for which j fires occurred (the F are necessary to perform the chi-square test). Since the number of fires occurring per time interval has no theoretical upper bound, the program assumes some upper limit s (currently it is 100) and checks to make sure that this limit is not exceeded by the data. If there are M time intervals altogether, by the relation between the F_j and the N_i we have for the first moment e_i of the data that

$$e_1 \equiv \frac{1}{M} \sum_{i=1}^{M} N_i$$

$$= \frac{1}{M} \sum_{j=0}^{s} jF_j$$

since the total number of fires which occurred is

$$\sum_{i=1}^{M} N_i = \sum_{j=0}^{s} jF_j$$

Similarly, for the second moment e_2 we have

$$e_2 \equiv \frac{1}{M} \sum_{i=1}^{M} N_i^2$$

$$= \frac{1}{M} \sum_{j=o}^{s} j^2 F_j$$

since F_j is the number of the N_i such that $N_i^2 = j^2$. Thus the program can use the equations involving the F_j's to calculate the first and second moments of the data.

The fact that there is no theoretical limit on how many fires may occur per time interval also affects how the chi-square cells are formed. The method of forming cells is as follows. The condition for forming a chi-square cells is that the expectation of the cell exceed some specified minimum (it is generally accepted that this minimum must be at least five). The expectation of a cell is the product of the total number of time intervals contained in the cell and the probability of the cell. The total number of time intervals is provided by the subroutine FRQNCY, so that dividing the minimum expectation by this number gives a minimum value for the probability of each cell. The program first looks at the number of time intervals for which no no fire occurred and computes their combined probability. This is just the probability of the frequency f_0 and is the expression for $P[N=n]$ (which is found in Appendix C-1) evaluated at $n=0$. If this probability exceeds the minimum required probability, then these time intervals form a cell. If the minimum is not exceeded, the program adds the time intervals with one fire occurring in them to the time intervals with no fires occurring in them. The probability of this sum is computed (it is just the sum of the probability of the frequency F_0 and the probability of the frequency F_1), and this new probability is then checks to see if it exceeds the minimum. If not, then the time intervals with two fires occurring in them are added to the time intervals with zero and one fires occurring in them, and so on until the combined probability of this sum is enough to form a cell. The other cells are formed in a similar manner, each starting with the lowest frequency which has not yet been used to form a cell.

In order to terminate this process, the total sum of the probabilities of all the cells is calculated after each cell is formed. Since the total

probability of all the frequencies 0,1,2,... must be one, cell formation can continue until one minus the total sum of the probabilities of all the cells is less than the minimum needed to form a cell. At this point, of course, no more cells can be formed, and the remaining frequency observations are added to the last cell formed. Since the condition for terminating this process is dependent upon the total probability of all the cells which have been formed (and hence upon the frequency observations themselves) one degree of freedom must be subtracted from the total number of degrees of freedom (which is the total number of cells formed). Also, since estimation of λ and μ requires two independent equations connecting the frequency observations (namely, the equations for e_1 and e_2), another two degrees of freedom must be subtracted. Because the chi-square test requires at least one net degree of freedom, at least four cells must be formed by the program to produce a result.

It is this requirement that prevents any results from being obtained. If the time interval is taken to be a week, the maximum number of cells which have been formed for any of the quarters is two. This is because of the skewed nature of the frequency distribution towards zero (see figure 1 of the main body of the report), and thus most of the probability is associated with the frequency of zero fires per time interval. As mentioned in the main body of the report, the probability associated with a frequency of zero fires per week for the third quarter is 73%, and this was the lowest probability associated with this frequency for all four quarters. It was shown in the main body of the report that the probabilities calculated by the computer for higher frequencies fall off dramatically for frequencies which are greater than $\hat{\mu}$, the estimated value for μ. Thus unless four

cells have been formed by the time the frequencies near $\hat{\mu}$ are used to form cells, the computer is unable to sum enough of the probabilities together to form a cell before an underflow condition occurs. This is in fact what happens, so that as a result we are unable to verify or reject the Poisson Batch model at this time.

A partial solution to this problem would be to devise some method of forming cells which does not depend upon one minus the total probability of the cells as a termination condition. This would then allow a requirement of only three cells to perform the test. As mentioned above, however, the greatest number of cells which have been formed by the program is two. Thus the only remaining alternative for the present model is to search for numerical methods which allow calculation of very small quantities. After comparison of figures 1 and 2 in the main body of the report, however, it might be more worthwhile to attempt to extend the Poisson model as was outlined in the main body of the report.

APPENDIX (C-3)

THE COMPUTER PROGRAM FOR TESTING THE POISSON BATCH MODEL

```
C       COMPUTER GENERATED FORTRAN TEXT
C       PROCESSED  9-MAY-79:  HMC ALTRAN V(2,7)A 2-FEB-79
C
C PROGRAM POSSON
C
C AUTHOR: MILTON SCRITSMIER
C
C DATE WRITTEN: MAY, 1979
C
C
C THIS PROGRAM TESTS THE POISSON BATCH MODEL
C FOR LIGHTNING AND ARSON CAUSED FIRES.
C
C INPUT REQUIRES A FILE ON DISK CALLED 'ZAP,BRN'
C WHICH CONTAINS THE FIRE TIMES (IN HOURS) IN SEQUENTIAL
C ORDER STARTING FROM TIME ZERO.  THIS FILE CAN BE
C OBTAINED BY USING THE SELECT PROGRAM WITH
C           'CAUSE= L'
C TO GET A FILE 'DATE.DAT' WHICH CONTAINS THE REQUIRED
C TIMES AND THEN TYPING THE COMMAND
C           'RENAME ZAP.BRN = DATE.DAT'
C TO THE MONITER DOT.
C
C BY ALTERING THE VARIABLES WHICH ARE TO BE FOUND IN THE
C BLOCK DATA SUBPROGRAM (WHICH IS LOCATED AT THE END OF
C THIS PROGRAM), ONE CAN CHANGE THE SIGNIFICANT PARAMETERS
C OF THE PROGRAM, WITH THE EXCEPTION THAT THE FIT TO THE
C POISSON BATCH MODEL IS ATTEMPTED ON A QUARTER BY QUARTER
C BASIS.  SEE THE BLOCK DATA SUBPROGRAM FOR A DESCRIPTION
C OF THESE PARAMETERS.
C
C IMPORTANT SYSTEM VARIABLES
C
C FREQ(I)   NUMBER OF TIME INTERVALS WITH
C           I FIRES OCCURING IN THEM
C           THE I TH TIME INTERVAL
C
C LAMDA     ESTIMATED POISSON PARAMETER FOR THE
C           OCCURRENCE OF LIGHTNING STORMS
C
C MU        ESTIMATED POISSON PARAMETER FOR THE
C           PROBABILITY OF A FIRE STARTING DUE TO
C           A LIGHTNING STORM
C
C INTNUM    TOTAL NUMBER OF TIME INTERVALS WITHIN
C           THE SPECIFIED QUARTERS OF ALL THE YEARS
C           FOR WHICH THERE IS DATA
C
C MAIN PROGRAM
C
C IMPORTANT VARIABLES
C
C CHISUM      CHI SQUARE STATISTIC
C
C X(I)        I TH CHI SQUARE CELL CONTAINING THE
C             GROUPED FREQUENCY OBSERVATIONS
C
C PRBSUM(I)   PROBABILITY OF THE FIRE OBSERVATIONS
C             IN X(I)
C
```

```
C MIN          MINIMUM EXPECTATION NECESSARY TO
C              FORM A CELL
C
C CELLNO       NUMBER OF COMPLETED CELLS
C
C DGFREE       DEGREES OF FREEDOM OF THE TEST
C
C
      DOUBLE PRECISION A, CHISUM, DIFSQR, TOTSUM,
     1                     PRBSUM(1:101), X(1:101)
      REAL LWRBND, MIN
      INTEGER DGFREE, LASTYR, ARRYNO, TOTFRQ, INTNUM, CELLNO,
     1           FREQ(0:100)
      COMMON FREQ /C/ ARRYNO /F/ INTNUM /I/ MIN
      DATA (X(I), I=0,100) /101*0/
      DATA (PRBSUM(I), I=0,100) /101*0/
      I = 0
      J = 1
      TOTSUM = 0
      TOTFRQ = 0
C
C DETERMINE FREQUENCIES AND ESTIMATE PARAMETERS
C
      CALL FRQNCY
      CALL PAREST
C
C SET UP CHI-SQUARE TEST BY FORMING CELLS
C
C STARTING WITH FREQ(0), ADD TOGETHER THE FREQUENCY
C VARIABLES FREQ(0), FREQ(1), FREQ(2), ..., , ALONG WITH
C ADDING TOGETHER THEIR CORRESPONDING PROBABILITIES
C UNTIL THE SUM OF THE PROBABILITIES ( REPRESENTED BY
C PRBSUM(J) ) IS GREATER THAN OR EQUAL TO LWRBND (LWRBND
C IS THE MINIMUM REQUIRED EXPECTATION OF THE CELLS
C DIVIDED BY THE NUMBER OF TIME INTERVALS IN A QUARTER).
C THE SUM OF THESE FREQUENCIES FORMS A CELL.  REPEAT
C THIS PROCESS ON THE REMAINING FREQUENCY VARIABLES
C UNTIL THE PROBABILITY OF THE REMAINING FREQUENCY
C VARIABLES IS TOO SMALL TO FORM A NEW CELL.  THESE
C REMAINING FREQUENCIES ARE THEN ADDED TO THE LAST
C CELL FORMED.
C
      LWRBND = MIN / INTNUM
   10 CONTINUE
      IF ((1 - TOTSUM) .LT. LWRBND) GO TO 50
   20 CONTINUE
      IF (PRBSUM(J) .GT. LWRBND) GO TO 40
      X(J) = X(J) + FREQ(I)
      PRBSUM(J) = PRBSUM(J) + PROB(I)
      I = I + 1
C
C CHECK FOR END OF RECORDED FREQUENCIES
C
      IF ( .NOT. (I .GT. ARRYNO) ) GO TO 30
      TYPE 100, CELLNO
      STOP
   30 CONTINUE
      GO TO 20
   40 CONTINUE
      TOTSUM = TOTSUM + PRBSUM(J)
```

```
      TOTFRQ = TOTFRQ + X(J)
      CELLNO = CELLNO + 1
      J = J + 1
      GO TO 10
   50 CONTINUE
C
C COLLAPSE REMAINING FREQUENCIES (IF ANY) INTO LAST CELL
C
      IF ( .NOT. (I .LE. ARRYNO) ) GO TO 60
      X(J-1) = X(J-1) + (INTNUM - TOTFRQ)
      PRBSUM(J-1) = PRBSUM(J-1) + (1 - TOTSUM)
   60 CONTINUE
C
C CHECK FOR FOUR OR MORE CELLS
C
      IF ( .NOT. (CELLNO .LT. 4) ) GO TO 70
C
C INSUFFICIENT NUMBER OF CELLS TO PERFORM TEST
C
      TYPE 100, CELLNO
      STOP
      GO TO 90
   70 CONTINUE
C
C SUFFICIENT NUMBER OF CELLS; CALCULATE CHI-SQUARE STATISTIC
C
      DO 80  K = 1,CELLNO
      DIFSQR = (X(K) - (INTNUM * PRBSUM(K))) ** 2
      CHISUM = CHISUM + (DIFSQR / (INTNUM * PRBSUM(K)))
   80 CONTINUE
      DGFREE = CELLNO - 3
      TYPE 110, CHISUM, DGFREE
   90 CONTINUE
      CALL EXIT
C
C FORMAT STATEMENTS
C
  100 FORMAT (/1X,'THE TEST CANNOT BE PERFORMED SINCE'/1X,
     1          'THERE ARE ONLY ',I1,' CELL(S).')
  110 FORMAT (/1X,'CHI-SQUARE = ', G16.5, //1X,
     1          'DEGREES OF FREEDOM = ', I2)
      STOP
      END
C
C
C THE FOLLOWING SUBROUTINE DETERMINES THE FREQUENCIES
C OF OBSERVATIONS FOR TIME INTERVALS OF FIXED LENGTH
C WITHIN A GIVEN QUARTER OF EACH YEAR FOR WHICH THERE
C IS DATA.
C
C IMPORTANT VARIABLES
C
C QTR      QUARTER OF THE YEAR FOR WHICH THE TEST
C          IS TO BE DONE
C
C BGNQTR   BEGINNING OF SPECIFIED QUARTER
C          FROM TIME ZERO
C
C ENDQTR   END OF SPECIFIED QUARTER FROM
C          TIME ZERO
```

```
C
C ENDTIM  END OF TIME INTERVAL FROM TIME ZERO
C
C FIRENO  NUMBER OF FIRES OBSERVED DURING A
C         TIME INTERVAL
C
C TIME    TIME FROM TIME ZERO OF THE
C         FIRE OBSERVATION
C
C TIMHRS  NUMBER OF HOURS IN EACH TIME INTERVAL
C
C
      SUBROUTINE FRQNCY
      INTEGER ARRYNO, QTR, BGNQTR, ENDQTR, ENDTIM, FIRENO, TIME,
     1        INTNUM, YEAR, LASTYR, LPYEAR, TIMHRS, YRHRS,
     2        QRTR(1:4), FREQ(0:100)
      REAL ADDYR, REYEAR
      COMMON FREQ /C/ ARRYNO /F/ INTNUM
     1        /G/ LASTYR /H/ LPYEAR /J/ TIMHRS
      INTNUM = 0
      SUPFRQ = 0
      ENDQTR = 0
      ENDTIM = 0
      QTR = 0
      YRHRS = 8760
      DATA (QRTR(I), I=1,4) /2160,2184,2208,2208/
C
C INPUT QUARTER OF THE YEAR FOR WHICH TEST IS TO BE PERFORMED
C
   10 CONTINUE
      IF ((QTR .GT. 0) .AND. (QTR .LT. 5)) GO TO 20
      TYPE 210
      ACCEPT 220, QTR
      GO TO 10
   20 CONTINUE
C
C CHECK TO SEE IF HOURS IN TIME INTERVAL EXCEED HOURS IN QUARTER;
C IF NOT, THEN OPEN INPUT FILE
C
      IF ( .NOT. (TIMHRS .GT. QRTR(QTR)) ) GO TO 30
      TYPE 230
      STOP
   30 CONTINUE
      OPEN (UNIT = 1, ACCESS = 'SEQIN', FILE = 'ZAP.BRN')
C
C CALCULATE QUARTER BOUNDS (UNCORRECTED FOR LEAP YEARS)
C
      DO 40  I = 1,QTR
      ENDQTR = ENDQTR + QRTR(I)
   40 CONTINUE
      BGNQTR = ENDQTR - QRTR(QTR)
C
C GET FIRST FIRE TIME
C
      READ (1, 240) TIME
C
C DETERMINE FREQUENCIES WITHIN GIVEN QUARTER OF EACH YEAR
C
      DO 150  YEAR = 0,LASTYR
C
```

```
C CORRECT QUARTER BOUNDS FOR LEAP YEARS
C
      REYEAR = YEAR
      ADDYR = ((REYEAR - LPYEAR) / 4) - ((YEAR - LPYEAR) / 4)
      IF ( .NOT. (ADDYR .EQ. Ø) ) GO TO 6Ø
      ENDQTR = ENDQTR + 24
      IF ( .NOT. (QTR .GT. 1) ) GO TO 5Ø
      BGNQTR = BGNQTR + 24
   5Ø CONTINUE
   6Ø CONTINUE
C
C DETERMINE END OF FIRST TIME INTERVAL OF THE QUARTER
C
      ENDTIM = BGNQTR + TIMHRS
C
C FIND BEGINNING OF QUARTER FOR GIVEN YEAR IN INPUT FILE
C
   7Ø CONTINUE
      IF (TIME .GT. BGNQTR) GO TO 8Ø
      READ (1, 24Ø, END = 17Ø) TIME
      GO TO 7Ø
   8Ø CONTINUE
C
C DETERMINE FREQUENCIES OF FIRES FOR EACH TIME INTERVAL IN
C GIVEN QUARTER OF THE GIVEN YEAR.
C
      FIRENO = Ø
   9Ø CONTINUE
C
C COUNT FIRES WHICH OCCURRED DURING CURRENT TIME INTERVAL
C
  1ØØ CONTINUE
      IF ( .NOT. (TIME .LE. ENDTIM) ) GO TO 11Ø
      FIRENO = FIRENO + 1
      READ (1, 24Ø, END = 16Ø) TIME
      GO TO 1ØØ
  11Ø CONTINUE
C
C INCREMENT COUNT OF TIME INTERVALS
C
      INTNUM = INTNUM + 1
C
C NOW; (1) CHECK IF FIRE COUNT EXCEEDS MAXIMUM ALLOWED FREQUENCY
C      (2) RECORD FIRE COUNT OF CURRENT TIME INTERVAL
C      (3) SET FIRE COUNT TO ZERO FOR NEXT TIME INTERVAL
C      (4) DETERMINE NEXT TIME INTERVAL
C      (5) CHECK IF NEW TIME INTERVAL EXTENDS PAST THE
C          END OF THE QUARTER
C
      IF ( .NOT. (FIRENO .GT. ARRYNO) ) GO TO 12Ø
      TYPE 25Ø, FIRENO
      STOP
  12Ø CONTINUE
      FREQ(FIRENO) = FREQ(FIRENO) + 1
      FIRENO = Ø
      ENDTIM = ENDTIM + TIMHRS
      IF (ENDTIM .GT. ENDQTR) GO TO 13Ø
C
C IF THE NEW TIME INTERVAL DOES NOT EXTEND PAST THE END OF THE
C QUARTER THEN REPEAT THIS PROCESS FOR THE NEW TIME INTERVAL
```

```
C
      GO TO 90
  130 CONTINUE
C
C  PREPARE QUARTER BOUNDS FOR NEXT YEAR
C
      BGNQTR = BGNQTR + YRHRS
      ENDQTR = ENDQTR + YRHRS
C
C IF THE QUARTER IS THE FIRST QUARTER, AND
C THE YEAR IS A LEAP YEAR, ADD A DAY TO
C THE BEGINNING OF THE QUARTER TO PREPARE
C FOR THE NEXT YEAR.
C
      IF ( .NOT. ((QTR .EQ. 1) .AND. (ADDYR .EQ. Ø)) ) GO TO 14Ø
      BGNQTR = BGNQTR + 24
  14Ø CONTINUE
  15Ø CONTINUE
      GOTO 19Ø
C
C END OF FILE; FINISH OFF QUARTER
C
C
C END OF FILE IN THE MIDDLE OF THE TIME INTERVAL;
C ASSUME DATA COMPLETE FOR THE TIME INTERVAL
C
  16Ø FREQ(FIRENO) = FREQ(FIRENO) + 1
      INTNUM = INTNUM + 1
      ENDTIM = ENDTIM + TIMHRS
C
C FINISH OFF QUARTER ASSUMING NO MORE FIRE
C OBSERVATIONS IN THE QUARTER
C
  17Ø CONTINUE
      IF (ENDTIM .GT. ENDQTR) GO TO 18Ø
      FREQ(Ø) = FREQ(Ø) + 1
      INTNUM = INTNUM + 1
      ENDTIM = ENDTIM + TIMHRS
      GO TO 17Ø
  18Ø CONTINUE
  19Ø CLOSE (UNIT = 1, FILE = 'ZAP.BRN')
C
C OUTPUT THE FREQUENCIES AND NUMBER OF TIME INTERVALS
C ALONG WITH THE NUMBER OF HOURS PER TIME INTERVAL
C
      TYPE 26Ø, QTR
      DO 2ØØ  I = Ø,ARRYNO - 1Ø,1Ø
      J = I
      TYPE 27Ø, J, (FREQ(K), K=J,J+9)
  2ØØ CONTINUE
      TYPE 27Ø, ARRYNO, FREQ(ARRYNO)
      TYPE 28Ø, INTNUM, TIMHRS
C
C FORMAT STATEMENTS
C
  21Ø FORMAT (/1X,'ENTER THE QUARTER OF THE YEAR AS 1,2,3, OR 4,'/)
  22Ø FORMAT (I1)
  23Ø FORMAT (/1X,'THE NUMBER OF HOURS IN THE TIME INTERVAL EXCEEDS'
     1          /1X,'THE NUMBER OF HOURS IN THE GIVEN QUARTER;'/
     2          /1X,'EXECUTION HALTED.')
```

```
  240 FORMAT (I6)
  250 FORMAT (/1X,'A FREQUENCY OF ' I3 ' WAS OBSERVED WHICH EXCEEDS'
     1        /1X,'THE DIMENSIONS OF THE VARIABLES FREQ AND X.'/
     2        /1X,'EXECUTION HALTED.')
  260 FORMAT (//T24'FREQUENCIES FOR QUARTER 'I1
     1        //T24'NUMBER OF TIME INTERVALS'
     2        /T26'WITH GIVEN FREQUENCY'///
     3        12X,'0',5X,'1',5X,'2',5X,'3',5X,'4',
     4        5X,'5',5X,'6',5X,'7',5X,'8',5X,'9',
     5        /'+'11X,'←',5X,'←',5X,'←',5X,'←',5X,'←',
     6        5X,'←',5X,'←',5X,'←',5X,'←',5X,'←'/)
  270 FORMAT (3X,I3,':',10I6)
  280 FORMAT (//1X,'THE NUMBER OF TIME INTERVALS IS ',I4
     1        /1X,'THE NUMBER OF HOURS PER TIME INTERVAL IS ',I4)
      RETURN
      END
C
C
C THE SUBROUTINE PAREST ESTIMATES THE PARAMETERS MU AND LAMDA
C BY USING THE METHOD OF MOMENTS
C
C IMPORTANT VARIABLES
C
C MOM1  FIRST MOMENT OF THE OBSERVATIONS
C
C MOM2  SECOND MOMENT OF THE OBSERVATIONS
C
C
      SUBROUTINE PAREST
      DOUBLE PRECISION MU, LAMDA, MOM1, MOM2, SUM, SUMSQ
      INTEGER ARRYNO, INTNUM, FREQ(0:100)
      COMMON FREQ /A/ MU, LAMDA /C/ ARRYNO /F/ INTNUM
      SUM = 0
      SUMSQ = 0
C
C GENERATE FIRST AND SECOND MOMENTS OF THE FREQUENCIES
C
      DO 10  I = 0,ARRYNO
      SUM = SUM + (I * FREQ(I))
      SUMSQ = SUMSQ + ((I ** 2) * FREQ(I))
   10 CONTINUE
      MOM1 = SUM / INTNUM
      MOM2 = SUMSQ / INTNUM
C
C CHECK IF THE PARAMETERS CAN BE CALCULATED
C
      IF ( .NOT. (MOM1 .EQ. 0) ) GO TO 20
      TYPE 30
      STOP
   20 CONTINUE
C
C CALCULATE MU AND LAMDA
C
      MU = (MOM2 / MOM1) - (MOM1 + 1)
      LAMDA = MOM1 / MU
C
C OUTPUT MU AND LAMDA OR AN ERROR MESSAGE
C
      TYPE 40, LAMDA, MU
   30 FORMAT (/1X,'THE FIRST MOMENT IS ZERO, AND THEREFORE THE'
```

```
     1          /1X,'PARAMETERS CANNOT BE ESTIMATED.'/
     2          /1X,'EXECUTION HALTED.')
   50 FORMAT (/1X,'THE PARAMETERS HAVE BEEN ESTIMATED, THEY ARE:'
     1          //1X,'LAMDA = ', G16.5 //1X,'MU = ', G16.5 /)
      RETURN
      END
C
C
C THE FOLLOWING FUNCTION GENERATES THE MOMENT GENERATING FUNCTION
C AND ITS DERIVATIVES AND IS USED TO FIND THE PROBABILITIES OF
C THE FREQUENCIES. IT IS ASSUMED IN ORDER TO CALCULATE THE
C COEFFICIENTS OF THE DERIVATIVES THAT CALLS TO MOMGEN START
C WITH N = 0 AND PROCEED SEQUENTIALLY TO N = 1,2,3, ... .
C
C IMPORTANT VARIABLES
C
C B         VALUE OF B(T), WHERE B(T) IS
C           THE FUNCTION ASSOCIATED WITH
C           THE MOMENT GENERATING FUNCTION
C
C N         NUMBER OF TIMES THE MOMENT GENERATING
C           FUNCTION IS TO BE DIFFERENTIATED
C
C T         PARAMETER OF THE MOMENT GENERATING
C           FUNCTION
C
C COEF(I)   I TH COEFFICIENT ASSOCIATED WITH THE
C           MOMENT GENERATING FUNCTION FOR A GIVEN N
C
C TEMP(I)   TEMPORARY STORAGE OF THE OLD VALUES OF
C           COEF(I) SO NEW VALUES CAN BE CALCULATED
C
C
      DOUBLE PRECISION FUNCTION MOMGEN (N,T)
      DOUBLE PRECISION  MU, LAMDA, B, SUM, ARG1, ARG2, LNMGEN
      REAL T
      INTEGER N, EXPLIM, COEF(1:100), TEMP(1:100)
      COMMON /A/ MU, LAMDA /B/ COEF /C/ ARRYNO /D/ EXPLIM
      SUM = 0
      ARG1 = MU * (T - 1)
C
C CHECK FOR UNDERFLOW IN CALCULATING EXPONENTIAL OF ARG1
C
      IF ( .NOT. (ARG1 .LT. EXPLIM) ) GO TO 10
      B = 0
      GO TO 20
   10 CONTINUE
      B = LAMDA * DEXP(ARG1)
   20 CONTINUE
      ARG2 = B - LAMDA
C
C CHECK FOR UNDERFLOW IN CALCULATING EXPONENTIAL OF ARG2
C
      IF ( .NOT. (ARG2 .LT. EXPLIM) ) GO TO 30
      MOMGEN = 0
      GO TO 120
   30 CONTINUE
C
C NO UNDERFLOW; CALCULATE MOMENT GENERATING FUNCTION
C OR A DERIVATIVE
```

```
C
      MOMGEN = DEXP(ARG2)
      IF ( .NOT. (N .GT. Ø) ) GO TO 11Ø
C
C A DERIVATIVE IS REQUIRED; CALCULATE COEFFICIENTS
C
      IF ( .NOT. (N .GT. 2) ) GO TO 6Ø
C
C INTIAL VALUES OF COEF(I) NO LONGER PROVIDE COEFFICIENTS;
C CALCULATE COEFFICIENTS FROM PREVIOUS COEFFICIENTS
C
      COEF(1) = 1
      COEF(N) = 1
      DO 4Ø  I = 1,ARRYNO
      TEMP(I) = COEF(I)
   4Ø CONTINUE
      DO 5Ø  I = 2,N-1
      COEF(I) = (I * TEMP(I)) + TEMP(I-1)
   5Ø CONTINUE
   6Ø CONTINUE
C
C N > Ø; CALCULATE A DERIVATIVE OF THE MOMENT GENERATING FUNCTION
C
      DO 8Ø  K = N,1,-1
C
C CALCULATE THE POLYNOMIAL IN B,
C CHECKING EACH POWER OF B FOR UNDERFLOW
C
      IF ( .NOT. ((K * DLOG(B)) .GE. EXPLIM) ) GO TO 7Ø
      SUM = SUM + (COEF(K) * (B ** K))
   7Ø CONTINUE
   8Ø CONTINUE
C
C NOW CHECK TO SEE IF THE POWER OF MU EXCEEDS
C THE UPPER LIMIT OF THE COMPUTER; IF SO
C THEN DO CALCULATION OF MOMGEN BY LOGS
C
      IF ( .NOT. ((N * DLOG(MU)) .GT. -EXPLIM) ) GO TO 9Ø
      LNMGEN = (DLOG(MOMGEN) + DLOG(SUM)) + N * DLOG(MU)
      MOMGEN = DEXP(LNMGEN)
      GO TO 1ØØ
   9Ø CONTINUE
      MOMGEN = ((MU ** N) * SUM) * MOMGEN
  1ØØ CONTINUE
  11Ø CONTINUE
  12Ø CONTINUE
      RETURN
      END
C
C
C THE FOLLOWING FUNCTION GENERATES THE PROBABILITIES
C FROM THE FREQUENCIES AND THE MOMGEN FUNCTION
C
C IMPORTANT VARIABLE
C
C I   FREQUENCY FOR WHICH PROBABILITY
C     IS BEING CALCULATED
C
C
      DOUBLE PRECISION FUNCTION PROB(I)
```

```
      DOUBLE PRECISION MOMGEN, LNFACT, LNARG, FACT, ARG
      INTEGER I, FACBND, EXPLIM
      COMMON /D/ EXPLIM /E/ FACBND
      EXTERNAL MOMGEN
C
C DETERMINE IF PROB(I) CAN BE CALCULATED DIRECTLY
C
      IF ( .NOT. (I .LE. FACBND) ) GO TO 30
C
C DIRECT CALCULATION
C
      FACT = 1
      IF ( .NOT. (I .GT. 0) ) GO TO 20
      DO 10  K = 1,I
      FACT = FACT * K
   10 CONTINUE
   20 CONTINUE
      PROB = MOMGEN(I,0.) / FACT
      GO TO 80
   30 CONTINUE
C
C INDIRECT CALCULATION USING LOGS
C
      LNFACT = 0
      IF ( .NOT. (I .GT. 0) ) GO TO 50
      DO 40  K = 1,I
      ARG = K
      LNFACT = LNFACT + DLOG(ARG)
   40 CONTINUE
   50 CONTINUE
      LNARG = DLOG(MOMGEN(I,0.)) - LNFACT
C
C CHECK TO SEE IF UNDERFLOW OCCURS FOR EXPONENTIAL
C OF LNARG; OTHERWISE CALCULATE PROB(I)
C
      IF ( .NOT. (LNARG .LT. EXPLIM) ) GO TO 60
      PROB = 0
      GO TO 70
   60 CONTINUE
      PROB = DEXP(LNARG)
   70 CONTINUE
   80 CONTINUE
      RETURN
      END
C
C
C THIS SUBPROGRAM INITIALIZES THE DATA IN COMMON
C
C THE PROGRAM PARAMETERS ARE:
C
C  ARRYNO  LARGEST NUMBER OF FIRES EXPECTED DURING ANY OF
C          THE TIME INTERVALS WITHIN THE QUARTER
C          (NOTE: THE UPPER LIMIT ON THE DIMENSIONING OF
C                 THE VARIABLES 'FREQ' AND 'X' IN THE MAIN
C                 PROGRAM, OF 'FREQ' IN THE SUBROUTINES
C                 FRQNCY AND PAREST, OF 'COEF' AND 'TEMP' IN
C                 THE FUNCTION MOMGEN, AND OF 'FREQ' AND 'COEF'
C                 IN THE BLOCK DATA SUBPROGRAM BELOW, MUST
C                 CORRESPOND TO ARRYNO.  IN THE DATA STATEMENTS
C                 BELOW, 'FREQ' MUST BE INITIALIZED TO 0,0,...,0,
```

```
C                   WHILE 'COEF' MUST BE INITIALIZED TO 1,1,0,0,
C                   ...,0,  FOR OUTPUT PURPOSES, ARRYNO SHOULD BE
C                   A MULTIPLE OF TEN.)
C
C   EXPLIM  LEAST INTEGER FOR WHICH EXPONENTIAL EXISTS
C           WITHOUT UNDERFLOW
C           (NOTE: THE NEGATIVE OF EXPLIM ALSO SERVES
C                   AS THE MAXIMUM INTEGER FOR WHICH THE
C                   EXPONENTIAL CAN BE COMPUTED WITHOUT
C                   OVERFLOW)
C
C   FACBND  LARGEST POSITIVE INTEGER FOR WHICH FACTORIAL
C           CAN BE COMPUTED WITHOUT OVERFLOW
C
C   LASTYR  LAST YEAR FROM TIME ZERO FOR WHICH DATA EXISTS
C
C   LPYEAR  FIRST LEAP YEAR FROM YEAR ZERO
C
C   MIN     THE MINIMUM EXPECTATION OF THE CHI-SQUARE CELLS
C
C   TIMHRS  THE LENGTH OF THE TIME INTERVAL IN HOURS
C
C ALL THE ABOVE PARAMETERS ARE OF INTEGER TYPE EXCEPT MIN,
C WHICH IS OF REAL TYPE
C
C
      BLOCK DATA
      INTEGER COEF(1:100), FREQ(0:100),
     1         ARRYNO, EXPLIM, FACBND,
     2         LASTYR, LPYEAR, TIMHRS
      REAL MIN
      COMMON FREQ /B/ COEF
     1        /C/ ARRYNO /D/ EXPLIM /E/ FACBND
     2        /G/ LASTYR /H/ LPYEAR /I/ MIN /J/ TIMHRS
      DATA COEF(1), COEF(2) /2*1/
      DATA (COEF(I), I=3,100) /98*0/
      DATA (FREQ(I), I=0,100) /101*0/
      DATA ARRYNO /100/
      DATA EXPLIM /-85/
      DATA FACBND /33/
      DATA LASTYR /7/
      DATA LPYEAR /2/
      DATA MIN /5./
      DATA TIMHRS /168/
      END
```

APPENDIX (C-4)

THE PROGRAM EXPFIT
AND THE KOLMOGOROV-SMIRNOV TESTS

The program EXPFIT was written to test the inter-fire times for an exponential distribution on a quarter by quarter basis by using both the standard Kolmogorov-Smirnov (K-S) and modified K-S tests which are described in the main body of the report. Input requires a file containing the ordered inter-fire times of the desired quarter for which the test is to be done. This file is obtained by running the program SELECT. By entering different fire parameters (fire size, fuel type, etc.) in SELECT, and running EXPFIT with the data files so obtained, one is able to determine which (if any) fire parameters cause an exponential fit for the inter-fire times. To interpret the statistics so obtained, see [9] in the Bibliography.

The program itself is relatively straightforward. The only feature of note is the function DELTA, which computes

$$\text{DELTA} = \max\left(\left|\frac{i}{n} - F(T_i)\right|, \left|F(T_i) - \frac{i-1}{n}\right|\right)$$

where $F(T_i)$ is the exponential probability distribution evaluated at T_i, the i^{th} ordered inter-fire time, and n is the total number of fires observed during the quarter.

APPENDIX (D)

THE BREAKDOWN OF FOREST FIRES IN REGION 5, BY FOREST, FUEL, TYPE, SIZE, FIRE DANGER AND QUARTER OF THE YEAR

Forest : 1 (Angeles) Fuel: Type A

name of the Forest

man caused — lightning caused — the 4th quarter

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	
Low	A									
	B									
	C									
	D								(c)	
	E									
	F									
	G									
Total		(e-f)								
Medium	A									
	B									
	C				(a)					
	D									
	E									
	F									
	G									
Total										
High	A									
	B									
	C									
	D									
	E									
	F									(d)
	G									
Total										
Very High	A									
	B									
	C									
	D					(b)				
	E									
	F									
	G									
Total										
Extreme	A									
	B									
	C									
	D									
	E									
	F									
	G									
Total										(g-h)
Total				(i)	(J)					k | l

Forest : 1 (Angeles) Fuel: B

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	8		6		1				15	0
	B	1		2			1	1		4	1
	C	1								1	
	D										
	E										
	F										
	G										
Total		(10-0)		(8-0)		(1-1)		(1-0)		(20-1)	
Medium	A	1		7	1	5	8	1		14	9
	B	2		3		1	2			6	2
	C	1		1		1		1		4	
	D										
	E										
	F										
	G										
Total		(4-0)		(11-1)		(7-10)		(2-0)		(24-11)	
High	A	1		9	4	9	11			19	15
	B			7	2	5	2			12	4
	C			4		2				6	
	D					1				1	
	E					2				2	
	F										
	G										
Total		(1-0)		(20-6)		(19-13)		(0-0)		(40-19)	
Very High	A			9		13	2			22	2
	B			5		10	1			15	1
	C			3		2				5	
	D			1		2				3	
	E					1				1	
	F			1						1	
	G										
Total		(0-0)		(19-0)		(28-3)		(0-0)		(47-3)	
Extreme	A			5		7		2		14	
	B			2		7				9	
	C			2		2				4	
	D			3						3	
	E	1		1						2	
	F			2		1				3	
	G					4				4	
Total		(1-0)		(15-0)		(21-0)		(2-0)		(39-0)	
Total		16	0	73	7	76	27	5	0	170	34

Here are some examples from the sample form of the previous page:

(Forest: 1, Fuel type A):

(a) indicates the number of size C lightning-caused fires, when the fire danger was medium in the second quarter.

(b) indicates the number of size D, man-caused fires, when the fire danger was very high, in the third quarter.

(c) indicates the total number of size D, man-caused fires, when the fire danger was low, in all quarters.

(d) indicates the total number of size F lightning-caused fires, when the fire danger was high, in all quarters.

(e) indicates the total number of size man-caused fires, when the fire danger was low, in the first quarter.

(f) the same as (e) but for lightning-caused fires.

(g) indicates the total number of man-caused fires, when the fire danger was extreme, in all quarters.

(h) the same as (g) but for lightning-caused fires.

(i) indicates the total number of man-caused fires in the second quarter.

(j) the same as (i) but for lightning-caused fires.

(k) indicates the total number of man-caused fires (in Forest 1 -fuel type A).

(l) the same but for lightning-caused fires.

NOTE: each page indicates one type of fuel.

NOTE: blank spaces indicate no-fire.

NOTE: size, cause, fire danger have been defined in the Individual Fire Report Handbook. Fuel types are described in Appendix (F).

Forest : 1 (Angeles) Fuel: A

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	9		16	1	6	1	2		33	2
	B	4		1		2				7	0
	C	1								1	0
	D										
	E										
	F										
	G										
Total		(14 - 0)		(17 - 1)		(8 - 1)		(2-0)		41 - 2	
Medium	A	2		11	2	7	1	4		24	3
	B			2		4		1		7	
	C					1				1	
	D					1				1	
	E					1				1	
	F										
	G										
Total		(2-0)		(13-2)		(14 - 1)		(5-0)		34 - 3	
High	A			36	2	23	10			59	12
	B			10		8	3			18	3
	C			3		4				7	
	D										
	E			1						1	
	F										
	G										
Total		(0-0)		(50-2)		(35-13)		(0-0)		(85 - 15)	
Very High	A			31	2	32	7	5		68	9
	B			9		9				18	
	C			3		2				5	
	D			1						1	
	E			1						1	
	F			1						1	
	G										
Total		(0-0)		(46-2)		(43-7)		(5-0)		(94 - 9)	
Extreme	A			12	1	34		2		48	1
	B	1		10		7				18	
	C			2		4				6	
	D			1		1				2	
	E					3				3	
	F					2				2	
	G					1				1	
Total		(1-0)		(25-1)		(52-0)		(2-0)		(80-1)	
Total		17	0	151	8	152	22	14	0	334	30

Forest: 1 (Angeles) Fuel: C

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A							1		1	
	B										
	C										
	D										
	E										
	F										
	G										
Total								(1-0)		(1-0)	
Medium	A					1		1		2	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)		(1-0)		(2-0)	
High	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Very High	A			2		3				5	
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total				(3-0)		(3-0)				(6-0)	
Extreme	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Total		0	0	(6	0)	4	0	1	0		

Forest : 1(Angeles) Fuel: F

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A					1				1	
	B						1				1
	C										
	D										
	E										
	F										
	G										
Total						(1-1)				(1-1)	
Medium	A			1	2					1	2
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-2)						(1-2)	
High	A			5	2	1	3			6	5
	B						1				1
	C					1				1	
	D										
	E										
	F										
	G										
Total				(5-2)		(2-4)				(7-6)	
Very High	A				1	5	2			5	3
	B			1		3				4	
	C	1								1	
	D										
	E										
	F										
	G										
Total		(1-0)		(1-1)		(8-2)				(10-3)	
Extreme	A					2				2	
	B					4				4	
	C										
	D										
	E										
	F										
	G										
Total						(6-0)				(6-0)	
Total		1	0	7	5	17	7	0	0	25	12

Forest : 1 (Angeles) Fuel: G

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	6		5	2	2	2			13	4
	B										
	C			1						1	
	D										
	E										
	F										
	G										
Total		(6-0)		(6-2)		(2-2)				(14-4)	
Medium	A				4	2	1	3		5	5
	B										
	C										
	D										
	E										
	F										
	G										
Total				(0-4)		(2-1)		(3-0)		(5-5)	
High	A	1		5	11	6	20	1		13	31
	B			1						1	
	C										
	D										
	E										
	F					1				1	
	G										
Total		(1-0)		(6-11)		(7-20)		(1-0)		(15-31)	
Very High	A			5	9	8	4			13	13
	B	1		1		1	1			3	1
	C										
	D										
	E						1				1
	F										
	G										
Total		(1-0)		(6-9)		(9-6)				(16-15)	
Extreme	A	1				4				5	
	B					2				2	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)				(6-0)				(7-0)	
Total		9	0	18	26	26	29	4	0	57	55

Forest : 1 (Angeles) Fuel: H

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
High	A					2				2	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(2-0)				(2-0)	
Very High	A			1		1				2	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(1-0)				(2-0)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		0	0	2	0	3	0			$\boxed{5}$	$\boxed{0}$

Forest: 1 (Angeles) Fuel: I

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Very High	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		0	0	1	0	0	0	0	0	$\boxed{1}$	$\boxed{0}$

Forest : 1 (Angeles) Fuel: K

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Very High	A			1				1		2	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)				(1-0)		(2-0)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total				1	0			1	0	$\boxed{2}$	$\boxed{0}$

Forest : 1 (Angeles) Fuel: L

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1						1		2	
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)						(1-0)		(2-0)	
Medium	A	2								2	
	B										
	C										
	D										
	E										
	F										
	G										
Total		(2-0)								(2-0)	
High	A					1				1	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Very High	A					1				1	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Extreme	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Total		3	0	1	0	2	0	1	0	7	0

Forest : 1 (Angeles) Fuel: T

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		7		1				9	
	B			1						1	
	C			1						1	
	D										
	E										
	F										
	G										
Total		(1-0)		(9-0)		(1-0)				(11-0)	
Medium	A	1		4		1				6	
	B						2				2
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(4-0)		(1-2)				(6-2)	
High	A			7	4	10	7	1		18	11
	B			4	1	3	3			7	4
	C			2						2	
	D										
	E					1				1	
	F										
	G										
Total				(13-5)		(14-10)		(1-0)		(28-15)	
Very High	A			6		9	2	1		16	2
	B			5		1				6	
	C			1		2				3	
	D										
	E										
	F										
	G										
Total				(12-0)		(12-2)		(1-0)		(25-2)	
Extreme	A			2		6				8	
	B			2		1				3	
	C										
	D										
	E										
	F										
	G										
Total				(4-0)		(7-0)				(11-0)	
Total		2	0	42	5	35	14	2	0	81	19

Forest: 2 (Cleveland) Fuel: A

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	2		7		3	1	1		13	1
	B										
	C			1						1	
	D										
	E										
	F										
	G										
Total		(2-0)		(8-0)		(3-1)		(1-0)		(14-1)	
Medium	A	2		3	1	6	1			11	2
	B			6		1	3			7	3
	C					1				1	
	D			1						1	
	E										
	F										
	G										
Total		(2-0)		(10-1)		(8-4)				(20-5)	
High	A	1		42		21		1		65	
	B	1		11	2	8	4			20	6
	C			4		2	1			6	1
	D			2						2	
	E										
	F										
	G										
Total		(2-0)		(59-2)		(31-5)		(1-0)		(93-7)	
Very High	A	3		27	1	34	3	1		65	4
	B			12		14	2			26	2
	C	1		6		2				9	
	D			1						1	
	E			1						1	
	F										
	G										
Total		(4-0)		(47-1)		(50-5)		(1-0)		(102-6)	
Extreme	A			4		17	1			21	1
	B			9		9	1			18	1
	C			1		2	1			3	1
	D										
	E					1				1	
	F										
	G										
Total				(14.-0)		(29-3)				(43-3)	
Total		10	0	13.8	4	121	18	3	0	272	22

Forest: 2 (Cleveland) Fuel: B

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	3			1	1	1			4	2
	B	1			1					1	1
	C					2				2	
	D			1						1	
	E										
	F										
	G										
Total		(4-0)		(1-2)		(3-1)				(8-3)	
Medium	A	3		3	3	1	2			7	5
	B			1		2	1			3	1
	C			1						1	
	D			1						1	
	E										
	F										
	G										
Total		(3-0)		(6-3)		(3-3)				(12-6)	
High	A	2		10	1	12	4	1		25	5
	B			5		9	1	1		15	1
	C			5		1				6	
	D			2						2	
	E			1						1	
	F					1				1	
	G										
Total		(2-0)		(23-1)		(23-5)		(2-0)		(50-6)	
Very High	A			13		17	5	5		35	5
	B			10		8	2			18	2
	C			3		3	1			6	1
	D			1		3				4	
	E										
	F			1						1	
	G										
Total				(28-0)		(31-8)		(5-0)		(64-8)	
Extreme	A			2		8		1		11	
	B			2		5		1		8	
	C			2		1				3	
	D										
	E					2				2	
	F			1		1				2	
	G					3				3	
Total				(7-0)		(20-0)		(2-0)		(29-0)	
Total		9	0	65	6	80	17	9	0	163	23

Forest : 2 (cleveland) Fuel: C

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A						1				1
	B										
	C										
	D										
	E										
	F										
	G										
Total						(0-1)				(0-1)	
Medium	A	1								1	
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)								(1-0)	
High	A			1			1			1	1
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total				(2-0)		(0-1)				(2-1)	
Very High	A			1		2	2			3	2
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(2-2)				(3-2)	
Extreme	A						1				1
	B										
	C										
	D										
	E										
	F										
	G										
Total						(0-1)				(0-1)	
Total		1	0	3	0	2	5			6	5

Forest: 2 (Cleveland) Fuel: F

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A			1		1				2	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(1-0)				(2-0)	
Very High	A			1						1	
	B			1						1	
	C										
	D										
	E			1						1	
	F										
	G										
Total				(3-0)						(3-0)	
Extreme	A					2				2	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(2-0)				(2-0)	
Total		0	0	4	0	3	0	0	0	7	0

Forest: 2 (Cleveland) Fuel: G

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		2		1	2			4	2
	B	1		1						2	
	C										
	D										
	E										
	F										
	G										
Total		(2-0)		(3-0)		(1-2)				(6-2)	
Medium	A	2		1		1	13	1		5	13
	B			1	1					1	1
	C			1						1	
	D										
	E										
	F										
	G										
Total		(2-0)		(3-1)		(1-13)		(1-0)		(7-14)	
High	A	1		12	2	5	9			18	11
	B			3						3	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(15-2)		(5-9)				(21-11)	
Very High	A	1		6	1	11	1	1		19	2
	B			3		2				5	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(9-1)		(13-1)		(1-0)		(24-2)	
Extreme	A					4	1			4	1
	B					2				2	
	C										
	D										
	E										
	F										
	G										
Total						(6-1)				(6-1)	
Total		6	0	30	4	26	26	2	0	64	30

Forest : 2 (Cleveland) Fuel: H

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A										
	B	1		1						2	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(1-0)						(2-0)	
Very High	A	1				1				2	
	B	1								1	
	C										
	D										
	E										
	F										
	G										
Total		(2-0)				(1-0)				(3-0)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		3	0	1	0	1	0	0	0	$\boxed{5}$	$\boxed{0}$

Forest : 2 (Cleveland) Fuel: K

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A			2		1				3	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(2-0)		(1-0)				(3-0)	
Very High	A					1				1	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Extreme	A	1				1		1		3	
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)				(1-0)		(1-0)		(3-0)	
Total		1	0	2	0	3	0	1	0	7	0

Forest : 2 (Cleveland) Fuel: L

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1									
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A			1	2	1				2	2
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-2)		(1-0)				(2-2)	
Very High	A					1				1	
	B					1				1	
	C										
	D										
	E										
	F										
	G										
Total						(2-0)				(2-0)	
Extreme	A							1		1	
	B										
	C										
	D										
	E										
	F										
	G										
Total								(1-0)		(1-0)	
Total		0	0	1	2	3	0	1	0	5	2

Forest: 2 (Cleveland) Fuel: T

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	5								5	
	B							1		1	
	C			1						1	
	D										
	E										
	F										
	G										
Total		(5-0)		(1-0)				(1-0)		(7-0)	
Medium	A			3			1			3	1
	B	1		1						2	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(4-0)		(0-1)				(5-1)	
High	A	1		10		6				17	0
	B			5	2	2				7	2
	C			1		1				2	
	D										
	E			1						1	
	F										
	G										
Total		(1-0)		(17-2)		(9-0)				(27-2)	
Very High	A			2		9		1		12	
	B			2	1	3		1		6	1
	C					2				2	
	D			1						1	
	E			1						1	
	F										
	G										
Total				(6-1)		(14-0)		(2-0)		(22-1)	
Extreme	A					4				4	
	B					1		1		2	
	C					1				1	
	D										
	E										
	F					1				1	
	G										
Total						(7-0)		(1-0)		(8-0)	
Total		7	0	28	3	30	1	4	0	69	4

Forest: 7 (Los Padres) Fuel: A

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	5		10						15	
	B	2		8		3				13	
	C				1			1		1	1
	D										
	E										
	F										
	G										
Total		(7-0)		(18-1)		(3-0)		(1-0)		(29-1)	
Medium	A			12		11				23	
	B			8		2				10	
	C			1		3				4	
	D										
	E										
	F										
	G										
Total				(21-0)		(16-0)				(37-0)	
High	A	2		30		30	5			62	5
	B	1		14		17				32	
	C			4		3				7	
	D					1				1	
	E			1						1	
	F					2				2	
	G										
Total		(3-0)		(49-0)		(53-5)				(105-5)	
Very High	A	2		17	1	27	2	1		47	3
	B			7		18				25	
	C			2		3				5	
	D					1				1	
	E										
	F					1				1	
	G										
Total		(2-0)		(26-1)		(50-2)		(1-0)		(79-3)	
Extreme	A			2	1	12				14	1
	B			4		3	2			7	2
	C			2		2				4	
	D										
	E										
	F			1		1				2	
	G										
Total				(9-1)		(18-2)				(27-3)	
Total		12	0	123	3	140	9	2	0	277	12

Forest: 7 (Los Padres) Fuel: B

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	2		5		3	2	1		11	2
	B	2			1					2	1
	C										
	D	1								1	
	E										
	F										
	G										
Total		(5-0)		(5-1)		(3-2)		(1-0)		(14-3)	
Medium	A			1	2	5	1			6	3
	B			8	1	5	1			13	2
	C			2		1				3	
	D			3		1				4	
	E										
	F										
	G										
Total				(14-3)		(12-2)				(26-5)	
High	A	2		8		16	2	1		27	2
	B			4	1	6		1		11	1
	C			7	1	5				12	1
	D			2	1	3				5	1
	E										
	F										
	G					1				1	
Total		(2-0)		(21-3)		(31-2)		(2-0)		(56-5)	
Very High	A			3	1	14	6			17	7
	B	1		5		8				14	
	C			1		7				8	
	D										
	E			1						1	
	F										
	G			1		1				2	
Total		(1-0)		(11-1)		(30-6)				(42-7)	
Extreme	A			3		3				6	
	B			1	1	2				3	1
	C	1				2	1	1		4	1
	D										
	E										
	F					1				1	
	G					1				1	
Total		(1-0)		(4-1)		(9-1)		(1-0)		(15-2)	
Total		9	0	55	9	85	13	4	0	153	22

Forest : 7 (Los Padres) . Fuel: C

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		1		1		1		4	
	B			2						2	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(3-0)		(1-0)		(1-0)		(6-0)	
Medium	A			1		1	1			2	1
	B					1				1	
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(2-1)				(3-1)	
High	A				1	3	3			3	4
	B			4						4	
	C										
	D										
	E										
	F										
	G										
Total				(4-1)		(3-3)				(7-4)	
Very High	A			5		2	1			7	1
	B			1		1				2	
	C						1				1
	D										
	E										
	F										
	G										
Total				(6-0)		(3-2)				(9-2)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		1	0	14	1	9	6	1	0	$\boxed{25}$	$\boxed{7}$

Forest: 7 (Los Padres) Fuel: F

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A			1		1	1			2	1
	B			1						1	
	C										
	D										
	E			1						1	
	F										
	G										
Total				(3-0)		(1-1)				(4-1)	
Medium	A			3	4					3	4
	B						2				2
	C										
	D										
	E										
	F										
	G										
Total				(3-4)		(0-2)				(3-6)	
High	A				2		1				3
	B			1						1	
	C										
	D							1		1	
	E										
	F										
	G										
Total				(1-2)		(0-1)		(1-0)		(2-3)	
Very High	A				1		2				3
	B										
	C										
	D										
	E										
	F										
	G										
Total				(0-1)		(0-2)				(0-3)	
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		0	0	7	7	1	6	1	0	$\boxed{9}$	$\boxed{13}$

Forest: 7 (Los Padres) Fuel: G

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A			2	1		5			2	6
	B										
	C										
	D										
	E										
	F										
	G										
Total				(2-1)		(0-5)				(2-6)	
Medium	A			6	9	4	2			10	11
	B			1		1	1			2	1
	C					1				1	
	D										
	E										
	F										
	G										
Total				(7-9)		(6-3)				(13-12)	
High	A			8	5	4	2			12	7
	B			3	1	1	2			4	3
	C										
	D										
	E										
	F										
	G										
Total				(11-6)		(5-4)				(16-10)	
Very High	A			6	3	1	9			7	12
	B				1	1				1	1
	C										
	D										
	E										
	F										
	G										
Total				(6-4)		(2-9)				(8-13)	
Extreme	A			1		3				4	
	B					2				2	
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(5-0)				(6-0)	
Total		0	0	27	20	18	21	0	0	45	41

Forest: 7 (Los Padres) Fuel: H

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A				1						1
	B										
	C										
	D										
	E										
	F										
	G										
Total				(0-1)						(0-1)	
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Very High	A										
	B			1		1				2	
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(1-0)				(2-0)	
Extreme	A					1				1	
	B										
	C					1				1	
	D										
	E										
	F										
	G										
Total						(2-0)				(2-0)	
Total		0	0	1	1	3	0	0	0	4	1

Forest: 7 (Los Padres) Fuel: K

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Medium	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
High	A					1				1	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Very High	A					1		1		2	
	B										
	C										
	D										
	E										
	F										
	G										
Total						(1-0)		(1-0)		(2-0)	
Extreme	A										
	B										
	C					1				1	
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Total		0	0	2	0	3	0	1	0	6	0

Forest: 7 (Los Padres) Fuel: L

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A			1						1	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)						(1-0)	
Medium	A			3						3	
	B					1				1	
	C										
	D										
	E										
	F										
	G										
Total				(3-0)		(1-0)				(4-0)	
High	A			1		3				4	
	B					2				2	
	C			1						1	
	D										
	E										
	F										
	G										
Total				(2-0)		(5-0)				(7-0)	
Very High	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Total		0	0	6	0	6	0			12	0

Forest : 7 (Los Padres) Fuel: T

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1								1	
	B			1		1				2	
	C			1				1		2	
	D										
	E										
	F										
	G										
Total		(1-0)		(2-0)		(1-0)		(1-0)		(5-0)	
Medium	A			2		2	1			4	1
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total				(3-0)		(2-1)				(5-1)	
High	A			10	2	9	4			19	6
	B			2	2	2				4	2
	C			2		2				4	
	D										
	E										
	F					1				1	
	G										
Total				(14-4)		(14-4)				(28-8)	
Very High	A			2	1	6	3			8	4
	B			6		2	1			8	1
	C					1				1	
	D										
	E										
	F										
	G										
Total				(8-1)		(9-4)				(17-5)	
Extreme	A			2		6	2	1		9	2
	B					4				4	
	C					1				1	
	D										
	E										
	F										
	G										
Total				(2-0)		(11-2)		(1-0)		(14-2)	
Total		1	0	29	5	37	11	2	0	69	16

Forest :(12) (San Bernardino) Fuel: A

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		1		1	2	1		4	2
	B			1				1		2	
	C			1						1	
	D										
	E										
	F										
	G										
Total		(1-0)		(3-0)		(1-2)		(2-0)		(7-2)	
Medium	A	4		20	1	17	4	4		45	5
	B			2		2		1		5	
	C	1		1						2	
	D	1								1	
	E										
	F										
	G										
Total		(6-6)		(23-1)		(19-4)		(5-0)		(53-5)	
High	A	3		48	5	43	3	7		101	8
	B	2		14		7				23	
	C			2	1	1				3	1
	D			2		1				3	
	E										
	F										
	G										
Total		(5-0)		(66-6)		(52-3)		(7-0)		(130-9)	
Very High	A	1		30	5	22	1			53	6
	B			20	1	15	1	1		36	2
	C			7		2		1		10	
	D			2						2	
	E										
	F										
	G			1						1	
Total		(1-0)		(60-6)		(39-2)		(2-0)		(102-8)	
Extreme	A			19		32				51	
	B			11		13				24	
	C			4		2				6	
	D					2				2	
	E			1		2				3	
	F			1		1				2	
	G					1				1	
Total				(36-0)		(53-0)				(89-0)	
Total		13	0	188	13	164	11	16	0	381	24

Forest : 12 (San Bernardino) Fuel: B

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	3			1		2			3	3
	B	1		1						2	
	C			2						2	
	D										
	E										
	F										
	G										
Total		(4-0)		(3-1)		(0-2)				(7-3)	
Medium	A	1		3			1			4	1
	B			1	1		1			1	2
	C						1	1		1	1
	D										
	E										
	F										
	G										
Total		(1-0)		(4-1)		(0-3)		(1-0)		(6-4)	
High	A						8	7		7	8
	B	1			1		2			1	3
	C			2				2		4	
	D	1								1	
	E										
	F							1		1	
	G										
Total		(2-0)		(2-1)		(0-10)		(10-0)		(14-11)	
Very High	A	1		6	4		1	4		11	5
	B			4			2	1		5	2
	C							2		2	
	D			3						3	
	E										
	F						1	1		1	1
	G										
Total		(1-0)		(13-4)		(0-4)		(8-0)		(22-8)	
Extreme	A			1				5		6	
	B			3				6		9	
	C			1				1		2	
	D							3		3	
	E			1				1		2	
	F							1		1	
	G			1				9		10	
Total				(7-0)				(26-0)		(33-0)	
Total		8	0	29	7	0	19	45	0	82	26

Forest : 12 (San Bernardino) Fuel: C

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A	1		1		1	4	2		5	4
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(1-0)		(1-4)		(2-0)		(5-4)	
High	A			2	1	3				5	1
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total				(3-1)		(3-0)				(6-1)	
Very High	A			1		1				2	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(1-0)				(2-0)	
Extreme	A						1	1		1	1
	B					1				1	
	C										
	D										
	E										
	F										
	G										
Total						(1-1)		(1-0)		(2-1)	
Total		1	0	5	1	6	5	3	0	15	6

Forest :12 (San Bernardino) Fuel: F

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A				1	1	2			1	3
	B						1				1
	C										
	D										
	E										
	F										
	G										
Total				(0-1)		(1-3)				(1-4)	
Medium	A	1		3	3	2	7			6	10
	B			1		1				2	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(4-3)		(3-7)				(8-10)	
High	A			15	9	14	2			29	11
	B	1		4	2	2	3			7	5
	C										
	D										
	E										
	F					1				1	
	G										
Total		(1-0)		(19-11)		(17-5)				(37-16)	
Very High	A			4	1	3	2			7	3
	B	1		1			2			2	2
	C										
	D			1						1	
	E										
	F										
	G										
Total		(1-0)		(6-1)		(3-4)				(10-5)	
Extreme	A			2		3				5	
	B										
	C										
	D										
	E										
	F										
	G										
Total		-		(2-0)		(3-0)				(5-0)	
Total		3	0	31	16	27	19	0	0	61	35

Fore'st : 12 (San Bernardino) Fuel: G

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		6	14	4	22	2		13	36
	B				1						1
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(6-15)		(4-22)		(2-0)		(13-37)	
Medium	A	16		44	46	17	103	3		80	149
	B			4	2	1				5	2
	C				1	1				1	1
	D										
	E										
	F										
	G										
Total		(16-0)		(48-49)		(19-103)		(3-0)		(86-152)	
High	A	5		76	47	100	27			181	74
	B			5		6	1	1		12	1
	C					1				1	
	D										
	E										
	F					1				1	
	G										
Total		(5-0)		(81-47)		(108-28)		(1-0)		(195-75)	
Very High	A	1		29	10	20	7			50	17
	B			4	1	2	1			6	2
	C			1		1				2	
	D			1						1	
	E										
	F										
	G										
Total		(1-0)		(35-11)		(23-8)				(59-19)	
Extreme	A	1		6	1	20	2			27	3
	B			1						1	
	C										
	D			1						1	
	E										
	F			1						1	
	G										
Total		(1-0)		(9-1)		(20-2)				(30-3)	
Total		24	0	179	123	174	163	6	0	383	286

Forest : 12 (San Bernardino) Fuel: H

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A	3		6		4				13	
	B			3						3	
	C										
	D										
	E										
	F										
	G										
Total		(3-0)		(9-0)		(4-0)				(16-0)	
High	A	1		7		14	2	1		23	2
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(7-0)		(14-2)		(1-0)		(23-2)	
Very High	A			1	1	3				4	1
	B										
	C										
	D										
	E										
	F										
	G										
Total				(1-1)		(3-0)				(4-1)	
Extreme	A			1		1				2	
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total				(2-0)		(1-0)				(3-0)	
Total		4	0	19	1	22	2	1	0	46	3

Forest : 12 (San Bernardino) Fuel: I

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Medium	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
High	A	1								1	
	B			1						1	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(1-0)						(2-0)	
Very High	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total											
Extreme	A										
	B										
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(1-0)							
Total		1	0	1	0	0	0			2	0

Forest: 12 (San Bernardino) Fuel: K

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	2					1			2	1
	B										
	C										
	D										
	E										
	F										
	G										
Total		(2-0)				(0-1)				(2-1)	
Medium	A	4		11	3	4	2	3		22	5
	B			2						2	
	C										
	D										
	E										
	F										
	G										
Total		(4-0)		(13-3)		(4-2)		(3-0)		(24-5)	
High	A	2		20	2	26		4		52	2
	B					2				2	
	C										
	D										
	E										
	F										
	G										
Total		(2-0)		(20-2)		(28-0)		(4-0)		(54-2)	
Very High	A			4	1	1				5	1
	B										
	C										
	D										
	E										
	F										
	G										
Total				(4-1)		(1-0)				(5-1)	
Extreme	A			2		1				3	
	B										
	C										
	D										
	E										
	F										
	G										
Total				(2-0)		(1-0)				(3-0)	
Total		8	0	39	6	34	3	7	0	88	9

Forest: 12 (San Bernardino) Fuel: L

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A										
	B					1				1	
	C										
	D										
	E										
	F										
	G										
Total						(1-0)				(1-0)	
Medium	A	1		1			1	1		3	1
	B	1				2				3	
	C										
	D										
	E										
	F										
	G										
Total		(2-0)		(1-0)		(2-1)		(1-0)		(6-1)	
High	A	1		3		2				6	
	B			2		1				3	
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(5-0)		(3-0)				(9-0)	
Very High	A							1		1	
	B										
	C										
	D										
	E										
	F										
	G										
Total								(1-0)		(1-0)	
Extreme	A					1				1	
	B			1		1				2	
	C										
	D										
	E										
	F										
	G										
Total				(1-0)		(2-0)				(3-0)	
Total		3	0	7	0	8	1	2	0	20	1

Forest : 12 (San Bernardino) Fuel: T

Danger	Size	Q1 M	Q1 L	Q2 M	Q2 L	Q3 M	Q3 L	Q4 M	Q4 L	Total M	Total L
Low	A	1		3	1		2			4	3
	B						1				1
	C										
	D										
	E										
	F										
	G										
Total		(1-0)		(3-1)		(0-3)				(4-4)	
Medium	A	2		13	8	6	16	1		22	24
	B			1	1					1	1
	C										
	D										
	E										
	F										
	G										
Total		(2-0)		(14-9)		(6-16)		(1-0)		(23-25)	
High	A	1		28	11	41	7	2		72	18
	B			6	1	4	2			10	3
	C			1	1					1	1
	D										
	E										
	F					1				1	
	G										
Total		(1-0)		(35-13)		(46-9)		(2-0)		(84-22)	
Very High	A			16	1	10	5	1		27	6
	B					1				1	
	C			2						2	
	D										
	E										
	F										
	G										
Total				(18-1)		(11-5)		(1-0)		(30-6)	
Extreme	A			6	1	9				15	1
	B			2		1				3	
	C			1						1	
	D										
	E										
	F										
	G										
Total				(9-1)		(10-0)				(19-1)	
Total		4	0	79	25	73	33	4	0	160	58

APPENDIX (E)

TABLES

Table 1

Critical values of $\hat{D}_n$ corresponding to test significance level

Sample Size n	Significance Level .20	.15	.10	.05	.01
4	.300	.319	.352	.381	.417
5	.285	.299	.315	.337	.405
6	.265	.277	.294	.319	.364
7	.247	.258	.276	.300	.348
8	.233	.244	.261	.285	.331
9	.223	.233	.249	.271	.311
10	.215	.224	.239	.258	.294
11	.206	.217	.230	.249	.284
12	.199	.212	.223	.242	.275
13	.190	.202	.214	.234	.268
14	.183	.194	.207	.227	.261
15	.177	.187	.201	.220	.257
16	.173	.182	.195	.213	.250
17	.169	.177	.189	.206	.245
18	.166	.173	.184	.200	.239
19	.163	.169	.179	.195	.235
20	.160	.166	.174	.190	.231
25	.142	.147	.158	.173	.200
30	.131	.136	.144	.161	.187
Over 30	$\frac{.736}{\sqrt{N}}$	$\frac{.768}{\sqrt{N}}$	$\frac{.805}{\sqrt{N}}$	$\frac{.886}{\sqrt{N}}$	$\frac{1.031}{\sqrt{N}}$

Source: 9 (see Bibliography)

Table 2

Table of Critical Values of D

Sample Size N	Level of Significance for $D = \text{Max}\,|F^*(X) - S_N(X)|$				
	.20	.15	.10	.05	.01
3	.451	.479	.511	.551	.600
4	.896	.422	.449	.487	.548
5	.359	.382	.406	.442	.504
6	.331	.351	.375	.408	.470
7	.309	.327	.350	.382	.442
8	.291	.308	.329	.360	.410
9	.277	.291	.311	.341	.399
10	.263	.277	.295	.325	.380
11	.251	.264	.283	.311	.365
12	.241	.254	.271	.298	.351
13	.232	.245	.261	.287	.338
14	.224	.237	.252	.277	.326
15	.217	.229	.244	.269	.315
16	.211	.222	.236	.261	.306
17	.204	.215	.229	.253	.297
18	.199	.210	.223	.246	.289
19	.193	.204	.218	.239	.283
20	.188	.199	.212	.234	.278
25	.170	.180	.191	.210	.247
30	155	.164	.174	.192	.226
Over 30	$\frac{.86}{\sqrt{N}}$	$\frac{.91}{\sqrt{N}}$	$\frac{.96}{\sqrt{N}}$	$\frac{1.06}{\sqrt{N}}$	$\frac{1.25}{\sqrt{N}}$

Source: 6 (see Bibliography)

Table 4

Table 7.2 Percentiles of the distribution of S and differences of expected values of reduced extreme-value order statistics

n	i	$Ez_{i+1}-Ez_i$	0.75	0.80	0.85	0.90	0.95	0.99
3	1	1.216395						
	2	0.863046						
	3		0.75	0.79	0.84	0.90	0.95	0.99
4	1	1.150727						
	2	0.706698						
	3	0.679596	0.74	0.79	0.85	0.90	0.95	0.99
	4		0.50	0.55	0.60	0.67	0.76	0.89
5	1	1.115718						
	2	0.645384						
	3	0.532445	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.583273	0.50	0.56	0.61	0.68	0.77	0.89
	5		0.67	0.71	0.75	0.79	0.86	0.94
6	1	1.093929						
	2	0.612330						
	3	0.474330	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.442920	0.50	0.55	0.61	0.68	0.76	0.89
	5	0.522759	0.67	0.71	0.75	0.80	0.86	0.93
	6		0.54	0.57	0.61	0.66	0.73	0.84
7	1	1.079055						
	2	0.591587						
	3	0.442789	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.387289	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.387714	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.480648	0.54	0.58	0.62	0.67	0.74	0.85
	7		0.64	0.67	0.70	0.74	0.80	0.88
8	1	1.068252						
	2	0.577339						
	3	0.422889	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.356967	0.50	0.55	0.61	0.68	0.77	0.90
	5	0.334089	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.349907	0.54	0.58	0.62	0.67	0.74	0.85
	7	0.449338	0.64	0.67	0.70	0.74	0.80	0.89
	8		0.55	0.58	0.61	0.65	0.71	0.81

Source: 9 (see Bibliography)

n	i	$Ez_{i+1}-Ez_i$	0.75	0.80	0.85	0.90	0.95	0.99
9	1	1.060046						
	2	0.566942						
	3	0.409157	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.337763	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.304777	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.297949	0.54	0.58	0.62	0.67	0.75	0.85
	7	0.322189	0.63	0.67	0.70	0.74	0.80	0.89
	8	0.424958	0.55	0.58	0.61	0.66	0.72	0.82
	9		0.62	0.64	0.67	0.71	0.76	0.85
10	1	1.053606						
	2	0.559013						
	3	0.399100	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.324470	0.50	0.55	0.61	0.68	0.77	0.90
	5	0.286163	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.269493	0.54	0.58	0.62	0.68	0.75	0.85
	7	0.271645	0.63	0.67	0.71	0.75	0.81	0.89
	8	0.300869	0.55	0.58	0.62	0.66	0.72	0.81
	9	0.405316	0.62	0.65	0.68	0.71	0.76	0.85
	10		0.55	0.58	0.61	0.64	0.69	0.79
11	1	1.048411						
	2	0.552769						
	3	0.391410	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.314705	0.49	0.55	0.61	0.68	0.77	0.90
	5	0.273245	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.251386	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.243928	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.251548	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.283879	0.62	0.64	0.68	0.71	0.77	0.85
	10	0.389071	0.55	0.58	0.61	0.64	0.70	0.79
	11		0.60	0.63	0.65	0.69	0.74	0.82
12	1	1.044137						
	2	0.547721						
	3	0.385338	0.75	0.79	0.84	0.90	0.95	0.99
	4	0.307221	0.50	0.55	0.61	0.68	0.78	0.89
	5	0.263737	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.238797	0.54	0.58	0.62	0.67	0.74	0.85
	7	0.226264	0.64	0.67	0.70	0.75	0.81	0.89
	8	0.224477	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.235630	0.62	0.64	0.68	0.71	0.77	0.85
	10	0.269966	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.375356	0.60	0.63	0.66	0.69	0.74	0.82
	12		0.55	0.57	0.60	0.63	0.68	0.76
13	1	1.040555						
	2	0.543556						
	3	0.380417	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.301300	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.256437	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.229515	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.213966	0.64	0.67	0.71	0.75	0.81	0.90
	8	0.207205	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.209131	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.222667	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.258323	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.363582	0.55	0.57	0.60	0.64	0.68	0.76
	13		0.59	0.61	0.64	0.67	0.72	0.79

Table 7.2 (continued)

n	i	$Ez_{i+1}-Ez_i$	0.75	0.80	0.85	0.90	0.95	0.99
14	1	1.037513						
	2	0.540059						
	3	0.376352	0.75	0.79	0.85	0.90	0.95	0.99
	4	0.296496	0.49	0.54	0.61	0.68	0.77	0.90
	5	0.250650	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.222377	0.54	0.58	0.62	0.68	0.74	0.86
	7	0.204885	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.195165	0.55	0.58	0.62	0.66	0.73	0.82
	9	0.192209	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.196679	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.211875	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.248409	0.55	0.57	0.60	0.64	0.68	0.77
	13	0.353334	0.59	0.61	0.64	0.67	0.72	0.79
	14		0.55	0.57	0.59	0.62	0.67	0.75
15	1	1.034894						
	2	0.537085						
	3	0.372934	0.75	0.80	0.84	0.90	0.95	0.99
	4	0.292518	0.51	0.56	0.62	0.69	0.78	0.90
	5	0.245947	0.68	0.71	0.76	0.80	0.86	0.94
	6	0.216712	0.54	0.58	0.62	0.67	0.75	0.86
	7	0.197893	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.186266	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.180402	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.180072	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.186347	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.202727	0.55	0.57	0.60	0.64	0.68	0.77
	13	0.239842	0.59	0.62	0.64	0.67	0.72	0.79
	14	0.344309	0.55	0.57	0.60	0.63	0.67	0.75
	15		0.59	0.61	0.63	0.66	0.70	0.77
16	1	1.032617						
	2	0.534521						
	3	0.370021	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.289169	0.51	0.56	0.62	0.69	0.78	0.89
	5	0.242049	0.68	0.72	0.76	0.80	0.86	0.94
	6	0.212103	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.192338	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.179407	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.171667	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.168476	0.55	0.58	0.61	0.65	0.71	0.79
	11	0.170026	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.177619	0.55	0.58	0.60	0.64	0.69	0.77
	13	0.194859	0.60	0.62	0.64	0.68	0.72	0.80
	14	0.232350	0.55	0.57	0.60	0.63	0.67	0.75
	15	0.336283	0.59	0.61	0.63	0.66	0.70	0.77
	16		0.55	0.57	0.59	0.62	0.66	0.73

The χ^2 Distribution[a]

Degrees of Freedom \ Pb	.995	.990	.975	.950	.900	.750	.500	.250	.100	.050	.025	.010	.005
1	$392704 \cdot 10^{-10}$	$157088 \cdot 10^{-9}$	$982069 \cdot 10^{-9}$	$393214 \cdot 10^{-8}$	.0157908	.1015308	.454937	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	.0100251	.0201007	.0506356	.102587	.210720	.575364	1.38629	2.77259	4.60517	5.99147	7.37776	9.21034	10.5966
3	.0717212	.114832	.215795	.351846	.584375	1.212534	2.36597	4.10835	6.25139	7.81473	9.34840	11.3449	12.8381
4	.206990	.297110	.484419	.710721	1.063623	1.92255	3.35670	5.38527	7.77944	9.48773	11.1433	13.2767	14.8602
5	.411740	.554300	.831211	1.145476	1.61031	2.67460	4.35146	6.62568	9.23635	11.0705	12.8325	15.0863	16.7496
6	.675727	.872085	1.237347	1.63539	2.20413	3.45460	5.34812	7.84080	10.6446	12.5916	14.4494	16.8119	18.5476
7	.989265	1.239043	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.344419	1.646482	2.17973	2.73264	3.48954	5.07064	7.34412	10.2188	13.3616	15.5073	17.5346	20.0902	21.9550
9	1.734926	2.087912	2.70039	3.32511	4.16816	5.89883	8.34283	11.3887	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.15585	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.5489	15.9871	18.3070	20.4831	23.2093	25.1882
11	2.60321	3.05347	3.81575	4.57481	5.57779	7.58412	10.3410	13.7007	17.2750	19.6751	21.9200	24.7250	26.7569
12	3.07382	3.57056	4.40379	5.22603	6.30380	8.43842	11.3403	14.8454	18.5494	21.0261	23.3367	26.2170	28.2995
13	3.56503	4.10691	5.00874	5.89186	7.04150	9.29906	12.3398	15.9839	19.8119	22.3621	24.7356	27.6883	29.8194
14	4.07468	4.66043	5.62872	6.57063	7.78953	10.1653	13.3393	17.1170	21.0642	23.6848	26.1190	29.1413	31.3193
15	4.60094	5.22935	6.26214	7.26094	8.54675	11.0365	14.3389	18.2451	22.3072	24.9958	27.4884	30.5779	32.8013
16	5.14224	5.81221	6.90766	7.96164	9.31223	11.9122	15.3385	19.3688	23.5418	26.2962	28.8454	31.9999	34.2672
17	5.69724	6.40776	7.56418	8.67176	10.0852	12.7919	16.3381	20.4887	24.7690	27.5871	30.1910	33.4087	35.7185
18	6.26481	7.01491	8.23075	9.39046	10.8649	13.6753	17.3379	21.6049	25.9894	28.8693	31.5264	34.8053	37.1564
19	6.84398	7.63273	8.90655	10.1170	11.6509	14.5620	18.3376	22.7178	27.2036	30.1435	32.8523	36.1908	38.5822
20	7.43386	8.26040	9.59083	10.8508	12.4426	15.4518	19.3374	23.8277	28.4120	31.4104	34.1696	37.5662	39.9968
21	8.03366	8.89720	10.28293	11.5913	13.2396	16.3444	20.3372	24.9348	29.6151	32.6705	35.4789	38.9321	41.4010
22	8.64272	9.54249	10.9823	12.3380	14.0415	17.2396	21.3370	26.0393	30.8133	33.9244	36.7807	40.2894	42.7956
23	9.26042	10.19567	11.6885	13.0905	14.8479	18.1373	22.3369	27.1413	32.0069	35.1725	38.0757	41.6384	44.1813
24	9.88623	10.8564	12.4011	13.8484	15.6587	19.0372	23.3367	28.2412	33.1963	36.4151	39.3641	42.9798	45.5585
25	10.5197	11.5240	13.1197	14.6114	16.4734	19.9393	24.3366	29.3389	34.3816	37.6525	40.6465	44.3141	46.9278
26	11.1603	12.1981	13.8439	15.3791	17.2919	20.8434	25.3364	30.4345	35.5631	38.8852	41.9232	45.6417	48.2899
27	11.8076	12.8786	14.5733	16.1513	18.1138	21.7494	26.3363	31.5284	36.7412	40.1133	43.1944	46.9630	49.6449
28	12.4613	13.5648	15.3079	16.9279	18.9392	22.6572	27.3363	32.6205	37.9159	41.3372	44.4607	48.2782	50.9933
29	13.1211	14.2565	16.0471	17.7083	19.7677	23.5666	28.3362	33.7109	39.0875	42.5569	45.7222	49.5879	52.3356
30	13.7867	14.9535	16.7908	18.4926	20.5992	24.4776	29.3360	34.7998	40.2560	43.7729	46.9792	50.8922	53.6720
40	20.7065	22.1643	24.4331	26.5093	29.0505	33.6603	39.3354	45.6160	51.8050	55.7585	59.3417	63.6907	66.7659
50	27.9907	29.7067	32.3574	34.7642	37.6886	42.9421	49.3349	56.3336	63.1671	67.5048	71.4202	76.1539	79.4900
60	35.5346	37.4848	40.4817	43.1879	46.4589	52.2938	59.3347	66.9814	74.3970	79.0819	83.2976	88.3794	91.9517
70	43.2752	45.4418	48.7576	51.7393	55.3290	61.6983	69.3344	77.5766	85.5271	90.5312	95.0231	100.425	104.215
80	51.1720	53.5400	57.1532	60.3915	64.2778	71.1445	79.3343	88.1303	96.5782	101.879	106.629	112.329	116.321
90	59.1963	61.7541	65.6466	69.1260	73.2912	80.6247	89.3342	98.6499	107.565	113.145	118.136	124.116	128.299
100	67.3276	70.0648	74.2219	77.9295	82.3581	90.1332	99.3341	109.141	118.498	124.342	129.561	135.807	140.169

[a] The probability shown at the head of the column is the area in the right-hand tail. Example: With 4 degrees of freedom, a χ^2 value larger than 7.78 has a .1 probability.

Source. This table is abridged from E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians,* Vol. I (1954), pp. 130–131, with kind permission of the Syndics of the Cambridge University Press, publishers for the Biometrika Society.

Table 5

Source: 16 (see Bibliography)

APPENDIX (F)

FUEL TYPES

APPENDIX (F)

FUEL TYPES

BRUSH SERIES

	Entry	Code	
Sagebrush - Large - The denser sagebrush of NE plateau (Eastside) and in Southern California associated with this in Northern California	Sagebrush -----------	2000	T
Sagebrush low - The low black sage in "wet" flats of NE plateau. Little or no grass.	Sagebrush low -------	2100	T
Light Chamise - Non timber soils. Chamise and chaparral on recent burns or on such poor soil that height growth is retarded. Open ground between bushes. Easy to walk through.	Light Chamise -------	2200	A
Moderate Chamise and Chaparral - Non timber soils. Height 3 to 6 feet and crowns touching, somewhat difficult to walk through.	Mod. Chamise & Chap. -	2300	B
Heavy Chamise and Chaparral - Non timber soils. Old growth usually over 6 feet in height. Very difficult or impossible to walk through.	H. Chamise & Chap. ---	2400	B
Light Brush on timber soils. East to walk through.	Light Brush ----------	2500	T
Medium Brush on timber soils. Somewhat difficult to walk through.	Medium Brush ---------	2600	F
Heavy Brush on timber soils. Very difficult or impossible to walk through.	Heavy Brush ----------	2700	B

HARDWOOD SERIES:

	Entry	Code	
Hardwoods - Mature - Mature hardwoods dominant on area. Canopies closed. Stems clear. Light ground cover or under story. Little chopping needed in building fireline.	Hardwoods - Mature ---	3000	C

INDIVIDUAL FIRE REPORT HANDBOOK, FORM 5100-29

Item 31, Fuel Type Precailing on Area--Vicinity of Origin.

This is the fuel, or fuels that burned. Select one fuel type from the list. Enter the indicated entry. Code Fields 66-69.

Select the appropriate code for the given fuels.

Fuel Type Description	Entry	Code	1/ 1978 NFDRS Fuel Model
GRASS SERIES:			
Annual Grass - Annual grasses and associated weeds. Open range. Cures early in season.	Annual Grass --------	1000	A
Perennial Grass - and associated weeds. Cures late in season. Open range type.	Perennial Grass -----	1100	L
Meadow Grass - Grasses in mountain meadows and dry meadows of NE plateau.	Meadow Grass --------	1200	L
Brushy Herbs - Fern prairies and glades of North Coast range. Fern and weeds, some grasses present.	Brushy Herbs --------	1400	L
Grass Woodland - Annual grass ground cover under hardwood (White oak generally) and in openings.	Grass Woodland ------	1500	A
Grass-Sage - Annual grass inssage areas of sufficient volume to be the primary cause of fire spread.	Grass-Sage ----------	1600	T

1/ Cosensus of Ted Storey, Dick Chase, Stan Rapp and Jack Carter (R5).

	Entry	Code	
Hardwoods - Young dense - Young hardwood stands under 20 feet in height. Large number stems per acre. Difficult to walk through. Tan oak and madrone young stands typical.	Hardwoods - Young ----	3100	B
CONIFEROUS TIMBER SERIES:			
Mature timber - old growth with no understory. Any species.	Mature timber --------	4000	G
Mature timber - bear clover understory ground cover.	Mature timber --------	4100	C
Mature timber - mixed brush and reproduction understory.	Mature timber --------	4200	G
Young timber - 0" - 4" diameter (thicket)	Young timber ---------	4300	U
Young timber - 4" - 12" diameter pole stands, light understory and moderate litter.	Young timber ---------	4400	H
Young timber - 12" - 20" diameter pole stands, light understory and heavy litter.	Young timber ---------	4500	K
SLASH SERIES:			
Slash light - because of light cut or high degree of disposal. Under 10 years old. Volume from _____ to ______	Slash light ----------	5000	K
Slash light - over 10 years old. Volume from _____ to ______	Slash light ----------	5100	K
Slash medium - under 10 years old.	Slash medium ---------	5200	J
Slash medium - over 10 years old.	Slash medium ---------	5300	J
Slash heavy - under 10 years old. Volume from _____ to _____	Slash heavy ----------	5400	I
Slash heavy - over 10 years old. Volume from _____ to _____	Slash heavy ----------	5500	I

TSI SLASH SERIES:

	Entry	Code	
1 - 3 years old, 10-20 tons per acre.	TSI slash 1-3 --------	6000	K
4 - 7 years old, 10-20 tons per acre.	TSI slash 4-7 --------	6100	K
8 years or more, 10-20 tons per acre.	TSI slash 8 or more --	6200	K
1 - 3 years old, 21 or more tons per acre.	TSI slash 1-3 --------	6300	I
4 - 7 years old, 21 or more tons per acre.	TSI slash 4-7 --------	6400	I
8 years or more, 21 or more tons per acre.	TSI slash 8 or more --	6500	I

NON-FOREST FUEL SERIES:

	Entry	Code	
Other - Any non-Forest fuels such as dumps, vehicles, buildings sawdust piles, slabs, edgings. log deck, lumber piles, etc.	Other (specify) ------	7000	

Item 32. Cost Class.

ENTRY. Enter estimated FFF cost of suppressing the fire until declared out.
Code Field 70.

Costs	Enter Specific Figure	Code
$0-100	-------	1
101-500	-------	2
501-1,500	-------	3
1,501-5,000	-------	4
5,001-25,000	-------	5
Over 25,000	-------	6

Item 33, Location

Code all items a. thru f. If section, township, and range are not available, only latitude and longitude need be completed.

Indicate map scale. Identify point of origin on map by X and enter section number at center of section. For class C and D fires sketch identifying roads, topography, etc. and perimeter of fire. Sketch separate map for E and larger fires.

BIBLIOGRAPHY

1. Barlow, R. E. and Proschan, F. P., (1975), Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston, Inc.

2. Barlow R. E., Proschan, F. P. and Hunter, L. C., (1965), Mathematical Theory of Reliability, John Wiley and Sons.

3. Birnbaum, Z. W., (1952), "Numerical Tabulation of the Distribution of Kolmogorov's Statistics for Finite Sample Size", Journal of American Statistical Association, vol. 47, pp. 425-441.

4. Chernoff, H. and Lehmann, E. L., (1954), "The Use of Maximum Likelihood Estimates in X^2 Tests for Goodness-of-Fit", The Annals of Mathematical Statistics, vol 25, pp. 579-586.

5. Lilliefors, H., (1967), "On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown", Journal of American Statistical Association, vol. 62, pp. 399-402.

6. Lilliefors, H., (1969), "On the Kolmogorov-Smirnov Test for the Exponential Distributions with Means Unknown", Journal of American Statistical Association, vol. 64, pp. 387-389.

7. Lindgren, B. W., (1962), Statistical Theory, The McMillan Company.

8. Mann, N. R., (1974), Methods for Statistical Analysis of Reliability and Life Data, John Wiley and Sons, Inc.

9. Mann, N. R., Schafer, R. E., and Singpurwalla, N. D., (1974), Methods of Statistical Analysis of Reliability and Life Data, John Wiley and Sons, Inc.

10. Mann, N. R., Scheuer, E. M., and Fertig, K. W., (1973), "A New Goodness-of-Fit Test for the Two-Parameter Weibull or Extreme Value Distriubtion with Unknown Parameters", Communications in Statistics, vol. 2, pp. 383-400.

11. Massey, L. H., (1951), "The Kolomogorov-Smirnov Test for Goodness-of-Fit", Journal of American Statistical Association, vol. 46, pp. 68-78.

12. Mees, R. M., (1972), A Method of Fittig Distributions to Forest Fire Data, U.S. Department of Agriculture, Southwest Forest and Range Experiment Station, Berkeley, California.

13. Miller, L. H., (1956), Table for Percentage Points of Kolmogorov Statistics", Journal of American Statistical Association, vol. 51, pp. 111-121.

14. Mood, A. M., Graybill, F.A., and Boes, D. C., (1974), Introduction to the Theory of Statistics, McGraw-Hill Series in Probability and Statistics.

15. Siley, S. D., (1975), Statistical Inference, Chapman and Hall Ltd.

16. Theil, H., (1971), Principles of Econometrics, John Wiley and Sons, Inc.

17. Tsokos, C. P. and Shimi, I. N., (1977), The Theory and Applications of Reliability, vol. I and II, Academic Press, Inc.